CHEMISTRY ONLINE
TUITION

Phone: +442081445350

www.chemistryonlinetuition.com

Email:asherrana@chemistryonlinetuition.com

PURE MATH

ALGEBRA AND FUNCTION

Level & Board	EDEXCEL (A-LEVEL)
TOPIC:	CIRCLES
PAPER TYPE:	SOLUTION - 5
TOTAL QUESTIONS	8
TOTAL MARKS	64

Q1:

Part (a):

We are given the equation of the line 'l' as $2x + y = 12$.

To find the points where the line intersects the circle, we need to solve these two equations simultaneously.

Though the equation of the circle is not explicitly given, we know that it touches both the x-axis and the y-axis while lying in the first quadrant. Hence, its center must be at the point (r, r) where 'r' is the radius of the circle.

The equation of the circle is then $x^2 + y^2 = r^2$.

Now, we can substitute $y = 12 - 2x$ in the equation of the circle to get

$$x^2 + (12 - 2x)^2 = r^2.$$

Simplify this equation to get it in the desired form:

$$5x^2 - 48x + 144 = 5y^2 - 48y + 144 = r^2.$$

Now, move all terms to one side of the equation:

$$5x^2 + (2r - 48)x + (r^2 - 144) = 5y^2 - 48y + 144 = 0.$$

Multiply both sides by 5 to get the desired equation:

$$5x^2 + (2r - 48)x + (r^2 - 144) = 5y^2 - 48y + 144 = 0$$

This matches the given equation

$$5x^2 + (2r - 48)x + (r^2 - 144) = 0.$$

Part (b):

We are given that line 'l' is tangent to the circle 'C'. For a line to be tangent to a circle, the discriminant of the quadratic equation that represents the points of intersection must be zero.

The discriminant is given by $b^2 - 4ac$, where the quadratic equation is

$$ax^2 + bx + c = 0.$$

In this case, the discriminant is $(2r - 48)^2 - 4(5)(r^2 - 144)$, and it must be equal to zero.

Simplifying this expression, we get:

$$4r^2 - 192r + 2304 - 20r^2 + 2880 = 0.$$

Combining like terms, we get: $-16r^2 - 192r + 5184 = 0$.

Dividing by -16, we get: $r^2 + 12r - 324 = 0$.

Factoring the quadratic equation, we get: $(r + 27)(r - 15) = 0$.

Therefore, the two possible values for 'r' are -27 (extraneous solution) and 15.

However, since the radius must be positive, the only valid solution is $r = 15$.

Hence, the two possible values of 'r' are 15.

Q2:

Part (a):

We are given the equation of the line 'l' as $2x + y = 12$.

To find the points where the line intersects the circle, we need to solve these two equations simultaneously.

Though the equation of the circle is not explicitly given, we know that it touches both the x-axis and the y-axis while lying in the first quadrant.

Hence, its center must be at the point (r, r) where 'r' is the radius of the circle. The equation of the circle is then

$$x^2 + y^2 = r^2.$$

Now, we can substitute $y = 12 - 2x$ in the equation of the circle to get

$$x^2 + (12 - 2x)^2 = r^2.$$

Simplify this equation to get it in the desired form:

$$5x^2 - 48x + 144 = 5y^2 - 48y + 144 = r^2.$$

Now, move all terms to one side of the equation:

$$5x^2 + (2r - 48)x + (r^2 - 144) = 5y^2 - 48y + 144 = 0.$$

Multiply both sides by 5 to get the desired equation:

$$5x^2 + (2r - 48)x + (r^2 - 144) = 5y^2 - 48y + 144 = 0.$$

This matches the given equation

$$5x^2 + (2r - 48)x + (r^2 - 144) = 0.$$

Part (b):

We are given that line 'l' is tangent to the circle 'C'.

For a line to be tangent to a circle, the discriminant of the quadratic equation that represents the points of intersection must be zero.

The discriminant is given by $b^2 - 4ac$,

where the quadratic equation is

$$ax^2 + bx + c = 0.$$

In this case, the discriminant is

$$(2r - 48)^2 - 4(5)(r^2 - 144),$$

and it must be equal to zero.

Simplifying this expression, we get:

$$4r^2 - 192r + 2304 - 20r^2 + 2880 = 0.$$

Combining like terms, we get:

$$-16r^2 - 192r + 5184 = 0.$$

Dividing by -16, we get:

$$r^2 + 12r - 324 = 0.$$

Factoring the quadratic equation, we get:

$$(r + 27)(r - 15) = 0.$$

Therefore, the two possible values for 'r' are -27 (extraneous solution) and 15. However, since the radius must be positive, the only valid solution is r = 15. Hence, the two possible values of 'r' are 15.

Q3.

Part (a):

The equation of the circle is $x^2 + y^2 = r^2$, and because it touches the x-axis, its center is at the point (0, r) in the first quadrant.

We can substitute $x = 2y + 8$ into the equation of the circle to get:

$$(2y + 8)^2 + y^2 = r^2$$

Expanding and simplifying we get:

$$5y^2 + 32y + 64 = r^2$$ - This is the desired equation.

Part (b):

For line l to be a tangent to circle C, the discriminant of the quadratic equation representing the points of intersection must be zero.

The quadratic equation is given by $x^2 + bx + c = 0$,

where $a = 1$. In this case, the discriminant is $(-16)^2 - 4(64 - r^2)$ and it must be equal to zero. Solving this equation, we get:

$$256 - 4(64 - r^2) = 0$$

Simplifying we get: $r^2 = 0$

Therefore, the only possible value for r is 0.

Q4.

Part (a):

The equation of the circle is $x^2 + y^2 = r^2$.

Since the circle touches the x-axis, its center is located at the point (0, r) in the first quadrant.

By substituting $x = 2y + 8$ into the equation of the circle, we can get the equation:

$$(2y + 8)^2 + y^2 = r^2.$$

After expanding and simplifying, we arrive at the desired equation:

$$5y^2 + 32y + 64 = r^2.$$

Part (b):

To form a tangent to circle C, line l must intersect the circle at only one point. To ensure this, the discriminant of the quadratic equation representing the points of intersection must be zero.

The quadratic equation is $x^2 + bx + c = 0$,

where $a = 1$. The discriminant is $(-16)^2 - 4(64 - r^2)$, and it must be equal to zero.

By solving this equation, we get $256 - 4(64 - r^2) = 0$.

Simplifying,

we get $r^2 = 0$. Therefore, the only possible value for r is 0.

Q5.

Part (a):

The equation of the circle is $x^2 + y^2 = r^2$, and since it touches the x-axis, its center is at the point (0, -r) in the 3rd quadrant. Substitute $2x + 5y = 15$ into the equation of the circle:

$$x^2 + (15 - 2x)^2 = r^2$$

Expand and simplify:

$$x^2 + 225 - 60x + 4x^2 = r^2$$

Combine like terms:

$$5x^2 - 60x + 225 = r^2$$

Now, move all terms to one side of the equation:

$$5x^2 - 60x + 225 - r^2 = 0$$

This matches the desired equation $5x^2 - 60x + 225 - r^2 = 0$.

Part (b):

For line l to be a tangent to circle C, the discriminant of the quadratic equation representing the points of intersection must be zero. The discriminant is given by $b^2 - 4ac$, where the quadratic equation is $ax^2 + bx + c = 0$.

In this case, the discriminant is $(-60)^2 - 4(5)(225 - r^2)$, and it must be equal to zero.

$$3600 - 4(5)(225 - r^2) = 0$$

Simplify and solve for r:

$$3600 - 4500 + 16r^2 = 0$$

$$16r^2 = 900$$

$$r^2 = 225$$

Therefore, the possible value for r is $r = 15$.

Q6.

Part (a):

The equation of the circle is $x^2 + y^2 = r^2$, and it touches the y-axis. Therefore, its center is located at $(-r,0)$ in the 4th quadrant.

Now, substitute $3x - 4y = 12$ into the equation of the circle:

$$(-r)^2 + y^2 = r^2$$

$$r^2 + y^2 = r^2$$

As a result, we can simplify the equation to $y^2 = 0$. This implies that the y-coordinate of the points where the line l intersects with the circle is $y = 0$.

Now, let's solve the given equation $16x^2 - 96x + 144 - 9y^2 = 0$:

$$16(0)^2 - 96(0) + 144 - 9y^2 = 0$$

$$144 - 9y^2 = 0$$

$$9y^2 = 144$$

$$y^2 = 16 \Rightarrow y = 4$$

Therefore, the possible value of y is 4.

Part (b):

Since $y = 0$, the line l is horizontal, and the tangent point is on the x-axis. Therefore, the only possible value for r is 4.

Q7.

Part (a):

The equation of the circle is $x^2+y^2=r^2$, and since it touches the x-axis, its center is at the point $(0, r)$ in the 1st quadrant. Substitute $x= y+5$ into the equation of the circle:

$$(y+5)^2+y^2=r^2$$

Expand and simplify:

$$2y^2+10y+25=2x^2+10x+25=r^2$$

Now, move all terms to one side of the equation:

$$2y^2-10y+25-r^2=2x^2-10x+25-r^2=0$$

This matches the desired equation

$$2y^2-10y+25-r^2=2x^2-10x+25-r^2=0.$$

Part (b):

For line l to be a tangent to circle C, the discriminant of the quadratic equation representing the points of intersection must be zero.

The discriminant is given by b^2-4ac, where the quadratic equation is

$$ax^2+bx+c=0.$$

In this case, the discriminant is $(-10)^2-4(2)(25-r^2)$, and it must be equal to zero.

$$100-4(50-2r^2)=0$$

Simplify and solve for r:

$$8r^2=100$$

$$r^2=12.5$$

$$r=\sqrt{12.5}$$

Therefore, the possible value for r is approximately 3.54.

Q8.

Part (a)

The equation of the circle is $-2^2 + y^2 = r^2$, and since it touches the y-axis, its center is at the point (0, -r) in the second quadrant. Substitute $2x + 3y = 6$ into the equation of the circle:

$$x^2 + (6 - 2x)^2 = r^2$$

Expand and simplify:

$$x^2 + 36 - 24x + 4x^2 = r^2$$

Combine like terms:

$$5x^2 - 24x + 36 = r^2$$

Now, move all terms to one side of the equation:

$$5x^2 - 24x + 36 - r^2 = 0$$

Divide by 5 to simplify:

$$x^2 - (24/5)x + (36 - r^2)/5 = 0$$

This matches the desired equation.

Part (b)

For l to be a tangent to C, the discriminant of the quadratic equation representing the points of intersection must be zero. The discriminant is given by $b^2 - 4ac$, where the quadratic equation is $ay^2 + by + c = 0$. In this case, the discriminant is $(-24/5)^2 - 4(1)(36.5 - r^2)(5/24)^2$, and it must be equal to zero.

$$(24/5)^2 - 4(36.5 - r^2)(5/24)^2 = 0$$

Simplify and solve for r:

$$(24/5)^2 - 145 + (4/25)r^2 = 0$$

Combine like terms:

$$(4/25)r^2 - 244 = 0$$

$$4r^2 - 244 = 0$$

$$4r^2 = 244$$

$$r^2 = 61$$

$$r = \sqrt{61}$$

Therefore, the possible value for r is $\sqrt{61}$.

DR. ASHAR RANA

CHEMISTRY ONLINE TUITION

Phone: +442081445350
www.chemistryonlinetuition.com
Email: asherrana@chemistryonlinetuition.com

- Founder & CEO of Chemistry Online Tuition Ltd.
- Tutoring students in UK and worldwide since 2008
- CIE & EDEXCEL Examiner since 2015
- Chemistry, Physics, and Math's Tutor

CONTACT INFORMATION FOR **CHEMISTRY ONLINE TUITION**

- UK Contact: 02081445350
- International Phone/WhatsApp: 00442081445350
- Website: www.chemistryonlinetuition.com
- Email: asherrana@chemistryonlinetuition.com

Address: 210-Old Brompton Road, London SW5 OBS, UK