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PURE MATH

ALGEBRA AND FUNCTION

Level & Board	EDEXCEL (A-LEVEL)
TOPIC:	CIRCLES
PAPER TYPE:	SOLUTION - 2
TOTAL QUESTIONS	8
TOTAL MARKS	54

Q.1

(a) The equation of the circle is given as:

$$x^2 + y^2 - 6x + 10y + 9 = 0$$

To write this equation in the standard form

$$(x-h)^2 + (y-k)^2 = r^2 \quad \rightarrow (i)$$

Now,

$$x^2 - 6x + y^2 + 10y = -9$$

$$(x^2 - 6x + 9) + (y^2 + 10y + 25) = -9 + 9 + 25$$

$$(x-3)^2 + (y+5)^2 = 25 \quad \rightarrow (ii)$$

Now,

Comparing the equation (i) and (ii)

$\Longrightarrow \quad Center\ (h,k) = (3,5)$

$\Longrightarrow \quad radius\ (r) = 5$

So, the center of the circle is (3,-5) and the radius is 5.

(a) The line y = Kx iinrsects the circle at two distinct points.

Substituting this into the circle equation, we get

$$x^2 + (kx)^2 - 6x + 10(kx) + 9 = 0$$

$$x^2(1+k^2) - 6x + 10kx + 9 = 0$$

Here,

$$a = 1 + k^2, b = 4(2.5)k, c = 9$$

Now,

$$b^2 - 4ac \geq 0$$

The discriminant must be greater than or equal to zero

$$\left(4(2.5k)\right)^2 - 4(1+k^2)(a) \geq 0$$

$$100k^2 - 36 - 36k^2 \geq 0$$

$$64k^2 \geq 36$$

$$k^2 \geq \frac{9}{16}$$

$$k \geq \frac{3}{4}\ or\ k \leq \frac{-3}{4}$$

So, the range of values for k is

$$k \geq \frac{3}{4}\ or\ k \leq \frac{-3}{4}$$

Q.2

(a) To find the center, complete the square for both x and y

$$x^2 + 4x + y^2 - 6y - 3 = 0$$

$$x^2 + 4x + y^2 - 6y = 3$$

$$(x^2 + 4x + 4) + (y^2 - 6y + 9) = 3 + 4 + 9$$

$$(x + 2)^2 + (y - 3)^2 = 16$$

Comparing with the standard form, we get

$$center(h, k) = (-2,3)$$

So, the center of C is (-2,3)

(b) As

$$(x + 2)^2 + (y - 3)^2 = 16$$

OR

$$(x + 2)^2 + (y - 3)^2 = (4)^2 \rightarrow (i)$$

Using equation of circle

$$(x - h)^2 + (y - k)^2 = r^2 \rightarrow (ii)$$

Comparing equation (i) and (ii) with Rand hand side, we get

$$Radius\ (r) = 4$$

The radius of C is 4.

Q.3

(a) To find the center, complete the square for both x and y

$$\Longrightarrow \quad x^2 - 8x + y^2 + 6y = -16$$

$$\Longrightarrow \quad (x^2 - 8x + 16) + (y^2 + 6y + 9) = -16 + 16 + 9$$

$$\Longrightarrow \quad (x - 4)^2 + (y + 3)^2 = 9 \ \rightarrow (i)$$

Using standard form of the circle

$$(x - h)^2 + (y - k)^2 = r^2 \rightarrow (ii)$$

$$Center\ (h, k) = (4, -3)$$

(b) We are find the value of radius comparing (i) and (ii), we have

$$r = \sqrt{9}$$

$$r = \sqrt{3^2}$$

$$r = 3$$

So, the radius of A is 3.

Q.4

(a) Given equation of circle

$$x^2 - 8x + y^2 + 6y = -16$$

$$(x^2 - 8x + 16) + (y^2 + 6y + 9) = -16 + 16 + 9$$

$$(x-4)^2 + (y+3)^2 = 9$$

$$(x-4)^2 + (y+3)^2 = (3)^2 \rightarrow (i)$$

Using standard form of circle

$$(x-h)^2 + (y-k)^2 = (r)^2 \rightarrow (ii)$$

Comparing (i) and (ii) , we get

$$Center\ (h,k) = (4,-3)$$

(b) $radius\ (r) = 3$

(c) Consider the line y = kx , where k is a constant

Substituting y = kx into the circle equation

$$x^2 + (kx)^2 - 2x + 4(kx) + 5 = 0$$

$$x^2(1+k^2) + 2x(2k) + 5 = 0$$

For real solution , the discriminant must be greater than or equal to zero

$$\left(2(2k)\right)^2 - 4(1+k^2)(5) \geq 0$$

Simplify and solve for k:

$$16k^2 - 20(1+k^2) \geq 0$$

$$16k^2 - 20 - 20k^2 \geq 0$$

$$-4k^2 - 20 \geq 0$$

$$k^2 + 5 \leq 0$$

This inequality has no real solution for k, meaning that any line y = kx where

K is a constant will not intersect the circle C_4 , except the ease when the line

Coincides with the circle.

Q.5

The equation of circle by standard form is

$$\Longrightarrow \quad (x-h)^2 + (y-k)^2 = r^2$$

$\Longrightarrow \quad (x-5)^2 + \left(y-(-2)\right)^2 = (4)^2$

$\Longrightarrow \quad (x-5)^2 + (y+2)^2 = 16$

$\Longrightarrow \quad x^2 + 25 - 10x + y^2 + 4 - 4y - 16 = 0$

$\Longrightarrow \quad x^2 + y^2 - 10x - 4y + 29 - 16 = 0$

$\Longrightarrow \quad x^2 + y^2 - 10x - 4y + 13 = 0$

Q.6

The equation of circle by standard form is

$\Longrightarrow \quad (x-h)^2 + (y-k)^2 = r^2$

$\Longrightarrow \quad \left(x-\sqrt{2}\right)^2 + \left(y-\left(-3\sqrt{3}\right)\right)^2 = \left(2\sqrt{2}\right)^2$

$\Longrightarrow \quad x^2 + 2 - 2\sqrt{2}x + y^2 + 27 + 6\sqrt{3}y = 8$

$\Longrightarrow \quad x^2 + y^2 - 2\sqrt{2}x + 6\sqrt{3}y + 29 - 8 = 0$

$\Longrightarrow \quad x^2 + y^2 - 2\sqrt{2}x + 6\sqrt{3}y + 21 = 0$

Required equation of circle.

Q.7

We know that. The slope of line passing through two points (x_1, y_1) And (x_2, y_2) is given by:

$\Longrightarrow \quad m_{AB} = \frac{y_2 - y_1}{x_2 - x_1}$

$\Longrightarrow \quad m_{AB} = \frac{8-3}{7-2} = \frac{5}{5} = 1$

$\Longrightarrow \quad m_{CD} = \frac{y_2 - y_1}{x_2 - x_1}$

$\Longrightarrow \quad m_{CD} = \frac{4-(-1)}{-1-4} = \frac{5}{5} = 1$

$\Longrightarrow \quad m_{AB} = m_{CD}$

The opposite sides AB and CD are parallel

Q.8

(a) Using the equation of circle

$\Longrightarrow \quad (x-h)^2 + (y-k)^2 = r^2$

$\Longrightarrow \quad (x+3)^2 + (y-5)^2 = r^2$

(b) *Since, point A (4,2) lies on the circle, we can substitute these coordinates into*

the equation of the circle and solve for r:

$$(4+3)^2 + (2-5)^2 = r^2$$

$$(7)^2 + (-3)^{-2} = r^2$$

$$49 + 9 = r^2$$

$$r^2 = 58$$

Taking square root on B/S

$$\sqrt{r^2} = \sqrt{58}$$

$$r = \sqrt{58}$$

Required radius is $\sqrt{58}$

