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PURE MATH

ALGEBRA AND FUNCTION

Level & Board	EDEXCEL (A-LEVEL)
TOPIC:	DIFFERENTIATION
PAPER TYPE:	SOLUTION - 6
TOTAL QUESTIONS	8
TOTAL MARKS	43

Differentiation - 6

***i*)** $(\mathbf{1+2x})^{\mathbf{-1}}$

Solution:

$$(1+2x)^{-1} = (-1)(2x) + \frac{(-1)(-1-1)}{2!}(2x)^2 + \frac{(-1)(1-1)(1-12)}{3!}(2x)^3 + \cdots$$

$$= 1 + (-1)(2x) + \frac{(-1)(-2)(-3)}{6}8x^3 + \cdots$$

$$= 1 + (-1)(2x) + \frac{(-1)(-2)}{2}4x^2 + \frac{(-1)(-2)(-3)}{6}8x^3 + \cdots$$

$$(1+2x)^{-1} = 1 - 2x + 4x^2 - 8x^3 + \cdots \ \ is\ valid\ if\ |2x| < 1 \Longrightarrow |x| < \frac{1}{2}$$

***ii*)** $(\mathbf{1+x})^{\frac{\mathbf{-1}}{\mathbf{2}}}$

Solution:

$$(1+x)^{\frac{-1}{2}} = 1 + \left(-\frac{1}{3}\right)(x) + \frac{\left(-\frac{1}{3}\right)\left(-\frac{1}{3}-1\right)}{2\ \ !}(x)^2 + \frac{\left(-\frac{1}{3}\right)\left(-\frac{1}{3}-1\right)\left(-\frac{1}{3}-2\right)}{3\ \ !}(x)^3 + \cdots$$

$$= 1 + \left(-\frac{1}{3}\right)(x)\frac{\left(-\frac{1}{3}\right)\left(-\frac{4}{3}\right)}{2}x^2 + \frac{\left(-\frac{1}{3}\right)\left(-\frac{4}{3}\right)\left(-\frac{7}{3}\right)}{6}x^3 \ldots$$

$$= 1 + \left(-\frac{1}{3}\right)(x) + \frac{4}{9}.\frac{1}{2}x^2 - \frac{28}{27}.\frac{1}{6}x^3 + \cdots$$

$$(1+x)^{\frac{-1}{2}} = 1 - \frac{1}{3}x + \frac{2}{9}x^2 - \frac{14}{81}x^3 + \cdots \ \ is\ valid\ if\ |x| < 1$$

***iii*)** $(\mathbf{4-3x})^{\frac{\mathbf{1}}{\mathbf{2}}}$

Solution:

$$(4-3x)^{\frac{1}{2}} = 4^{\frac{1}{2}}\left(1 - \frac{3}{4}x\right)^{\frac{1}{2}}$$

$$= 2\left\{1 + \left(\frac{1}{2}\right)\left(-\frac{3}{4}x\right) + \frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2!!}\left(-\frac{3}{4}\right)^2 + \frac{\left(\frac{1}{2}\right)\left(\frac{3}{4}-1\right)\left(\frac{1}{2}-2\right)}{3!}\left(-\frac{3}{4}x\right)^3 + \cdots\right\}$$

$$= 2\left\{1 - \frac{3}{8}x + \frac{\left(\frac{1}{2}\right)\left(\frac{-1}{2}\right)}{2}\left(\frac{9}{16}x^2\right) - \frac{\left(\frac{1}{2}\right)\left(\frac{-1}{2}\right)\left(\frac{-3}{2}\right)}{6}\left(\frac{27}{64}x^3\right) + \cdots\right\}$$

$$= 2\left\{1 - \frac{3}{8}x - \frac{1}{4}.\frac{1}{2}\left(\frac{9}{16}x^2\right) - \frac{3}{8}.\frac{1}{6}.\left(\frac{27}{64}x^3\right) + \cdots\right\}$$

$$(4-3x)^{\frac{1}{2}} = 2\left\{1 - \frac{3}{8}x - \frac{9}{64}x^2 - \frac{27}{1024}x^3 + \cdots\right\}$$

$$= 2 - \frac{3}{4}x - \frac{9}{64}x^2 - \frac{27}{512}x^3 + \cdots \ \ is\ valid\ if\ \left|\frac{3}{4}x\right| < 1 \Longrightarrow |x| < \frac{4}{3}$$

iv) $\mathbf{(8-2x)^{-1}}$

Solution:

$$(8-2x)^{-1} = 8^{-1}\left(1-\frac{1}{4}x\right)^{-1}$$

$$= \frac{1}{6}\left\{1 + (-1)\left(-\frac{1}{4}x\right) + \frac{-1((-1-1)}{2!}\left(-\frac{1}{4}\right)^2 + \frac{-1(-1-1)(-1-2)}{3!}\left(-\frac{1}{4}x\right)^3 + \cdots\right\}$$

$$= \frac{1}{8}\left\{1 + \frac{1}{4}x + \frac{-1(-2)}{2}\left(\frac{1}{16}x^2\right) - \frac{-1(-2)(-3)}{6}\left(\frac{1}{16}x^2\right) + \cdots\right\}$$

$$(8-2x)^{-1} = \frac{1}{8}\left\{1 + \frac{1}{4}x + \frac{1}{16}x^2 + \frac{1}{64}x^3 + \cdots\right\}$$

$$= \frac{1}{8} + \frac{1}{32}x + \frac{1}{128}x^2 + \frac{1}{512}x^3 + \cdots is\ valid\ if\ \left|\frac{1}{4}x\right| < 1 \Longrightarrow |x| < 4$$

v) $\mathbf{(2-3x)^{-2}}$

Solution:

$$(2-3x)^{-2} = 2^{-2}\left(1-\frac{3}{2}x\right)^{-2}$$

$$= \frac{1}{4}\left\{1 + (-2)\left(-\frac{3}{2}x\right) + \frac{-2(-2-1)}{2!} + \left(-\frac{3}{2}\right)^2 + \frac{-2(-2-1)(-2-2)}{3!}\left(-\frac{3}{2}\right)^3 + \cdots\right\}$$

$$= \frac{1}{4}\left\{1 + 3x + \frac{-2(-3)}{2}\left(\frac{9}{4}x^2\right) - \frac{(-2)(-3)(-4)}{6}\left(\frac{27}{8}x^3\right) + \cdots\right\}$$

$$(2-3x)^{-2} = \frac{1}{4}\left\{1 + 3x + \frac{27}{4}x^2 + \frac{27}{2}x^3 + \cdots\right\}$$

$$= \frac{1}{4} + \frac{3}{4}x + \frac{27}{4}x^2 + \frac{27}{8}x^3 + \cdots \ \ is\ valid\ if\ \left|\frac{3}{2}x\right| < 1 \Longrightarrow |x| < \frac{2}{3}$$

vi) $\mathbf{\dfrac{(1-x)^{-1}}{(1+x)^2}}$

Solution:

$$\frac{(1-x)^{-1}}{(1+x)^2} = (1-x)^{-1}(1+x)^{-2}$$

$$= \left\{1 + (-1)(-x) + \frac{(-1)(-1-1)}{2!}(-x)^2 + \frac{(-1)(-1-1)(-1-2)}{3!}(-x)^2 + \cdots\right\}$$

$$\times\left\{1 + (-2)(x) + \frac{(-2)(-2-1)}{2!}(x)^2 + \frac{(-2)(-2-1)(-2-2)}{3!}(x)^3 + \cdots\right\}$$

$$= \left\{1 + x + \frac{(-1)(-2)}{2}x^2 - \frac{(-1)(-2)(-3)}{6}x^2 + \cdots\right\} \times \left\{1 - 2x + \frac{(-2)(-3)}{2}x^2 + \frac{(-2)(-3)(-4)}{6}x^3 + \ldots\right\}$$

$$= \{1 + x + x^2 + x^3 + \cdots\} \times \{1 - 2x + 3x^2 - 4x^3 + \cdots\}.$$

$$= 1 - 2x + 3x^2 - 4x^3 + x - 2x^2 + 3x^3 + x^2 - 2x^3 + x^3 + \cdots$$

$$\frac{(1-x)^{-1}}{(1+x)^2} = 1 - x + 2x^2 - 2x^3 + \cdots$$

The expression $(1-x)^{-1}$ *and* $(1+x)^{-2}$ *are valid if* $|x| < 1$

Those the expression $\frac{(1-x)^{-1}}{(1+x)^2}$ *is valid if* $|x| < 1$

vii) $\dfrac{\sqrt{1+2x}}{1-x}$

Solution:

$$\frac{\sqrt{1+2x}}{1-x} = (1+2x)^{\frac{1}{2}}(1-x)^{-1}$$

$$= \left\{1 + \left(\frac{1}{2}\right)(2x) + \frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2!}(2x^2) + \frac{\frac{1}{2}\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)}{3!}(2x^3) + \cdots\right\}$$

$$\times \left\{1 + (-1)(-x) + \frac{(-1)(-1-1)}{2!}(-x)^2 + \frac{(-1)(1-1)(-1-2)}{3!}(-x)^3 + \cdots\right\}$$

$$= \left\{1 + x + \frac{\frac{1}{2}\left(\frac{-1}{2}\right)}{2}(4x^2) + \frac{\frac{1}{2}\left(\frac{-1}{2}\right)\left(\frac{-3}{2}\right)}{6}(8x^3) + \cdots\right\} \times \left\{1 + x + \frac{(-1)(-2)}{2}x^2 - \frac{(-1)(-2)(-3)}{6}x^3 + \cdots\right\}$$

$$= \left\{1 + x - \frac{1}{4}, \frac{1}{2}(4x^2) + \frac{3}{8}.\frac{1}{6}(8x^3) + \cdots\right\} \times \{1 + x + x^2 + x^3 + \cdots\}$$

$$= \left\{1 + x - \frac{1}{2}x^2 + \frac{1}{2}x^3 + \cdots\right\} \times \{1 + x + x^2 + x^3 + \cdots\}$$

$$= 1 + x + x^2 + x^3 + x + x^2 + x^3 - \frac{1}{2}x^2 - \frac{1}{2}x^2 + \frac{1}{2}x^3 + \cdots$$

$$\frac{\sqrt{1+2x}}{1-x} = 1 + 2x + \left(1 + 1 - \frac{1}{2}\right)x^2 + 2x^3 + \cdots$$

$$= 1 + 2x + \frac{3}{2}x^2 + 2x^3 + \cdots$$

The expression $\sqrt{1+2x}$ *is valid of* $|2x| < 1 \implies |x| < \frac{1}{2}$ *and the expression* $(1-x)^{-1}$ *is valid if* $|x < 1|$

Thus the expression $\frac{\sqrt{1+2x}}{1-x}$ *is valid if* $|x| < \frac{1}{2}$

viii) $\dfrac{(4+2x)^{\frac{1}{2}}}{2-x}$

Solution:

$\frac{(4+2x)^{\frac{1}{2}}}{2-x}$ $= \sqrt{4}\left(1+\frac{X}{2}\right)^{\frac{1}{2}} 2^{-1}\left(1-\frac{X}{2}\right)^{-1}$

$= \left\{1+\left(\frac{1}{2}\right)\left(\frac{X}{2}\right)+\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2!}\left(\frac{X}{2}\right)^2+\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)}{3!}\left(\frac{X}{2}\right)^3+\cdots\right\}$

$\times\frac{1}{2}\left\{1+(-1)\left(-\frac{X}{2}\right)+\frac{(-1)(-1-)}{2!}\left(-\frac{X}{2}\right)^2+\frac{(-1)(-1-1)(-1-2)}{3!}\left(-\frac{X}{2}\right)^3+\cdots\right\}$

$= \left\{1+\frac{X}{4}+\frac{\frac{1}{2}\left(\frac{-1}{2}\right)}{2}+\frac{\frac{1}{2}\left(\frac{-1}{2}\right)\left(\frac{-3}{2}\right)}{6}\left(\frac{X^3}{8}\right)+\cdots\right\}\times\left\{1+\frac{X}{2}+\frac{(-1)(-2)}{2}\frac{x^2}{4}-\frac{(-1)(-2)(-3)}{6}\frac{x^3}{8}+\cdots\right\}$

$= \left\{1+\frac{x}{4}-\frac{1}{4}.\frac{1}{2}.\left(\frac{x^2}{4}\right)+\frac{3}{4}.\frac{1}{6}.\left(\frac{x^3}{8}\right)+\cdots\right\}\times\left\{1+\frac{x}{2}+\frac{x^2}{4}+\frac{x^3}{8}+\cdots\right\}$

$= \left\{1+\frac{x}{4}-\frac{x^2}{32}+\frac{x^3}{128}+\cdots\right\}\times\left\{1+\frac{x}{2}+\frac{x^2}{4}+\frac{x^3}{8}+\cdots\right\}$

$= 1+\frac{x}{2}+\frac{x^2}{4}+\frac{x^3}{8}+\frac{x}{4}+\frac{x^2}{8}+\frac{x^3}{16}-\frac{x^2}{32}-\frac{x^3}{64}+\frac{x^3}{128}+\cdots$

$= 1+\left(\frac{x}{2}+\frac{x}{4}\right)+\left(\frac{x^2}{4}+\frac{x^2}{8}-\frac{x^2}{32}\right)+\left(\frac{x^3}{8}+\frac{x^3}{16}-\frac{x^3}{64}+\frac{x^3}{128}\right)+\cdots$

$\frac{\sqrt{4+2x}}{2-x} = 1+\frac{3}{4}x+\frac{11}{32}x^2+\frac{23}{128}x^3\ldots$

$= The\ expression\ \sqrt{4+2x}\ is\ valid\ if\ \left|\frac{x}{2}\right|<1 \Rightarrow |x|<2$

$= and\ the\ expression\ (2-x)^{-1} is\ valid\ if\ \left|\frac{x}{2}\right|<1 \Rightarrow |x|<2$

$Thus\ the\ expression\ \frac{\sqrt{4+2x}}{2-x} is\ valid\ if\ |x|<2$

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