

Phone: +442081445350
www.chemistryonlinetuition.com

Email:asherrana@chemistryonlinetuition.com

## PURE MATH

## ALGEBRA AND FUNCTION

Level \& Board

TOPIC:
FACTOR THEOREM

PAPER TYPE:44

## Questions

Q1.

$$
f(x)=4 x^{3}-12 x^{2}+2 x-6
$$

(a) Use the factor theorem to show that $(x-3)$ is a factor of $\mathrm{f}(x)$.
(b) Hence show that 3 is the only real root of the equation $\mathrm{f}(x)=0$
(4)
(Total for question = 6 marks)

Q2.

$$
g(x)=4 x^{3}-12 x^{2}-15 x+50
$$

(a) Use the factor theorem to show that $(x+2)$ is a factor of $\mathrm{g}(x)$.
(2)
(b) Hence show that $\mathrm{g}(x)$ can be written in the form $\mathrm{g}(x)=(x+2)(\mathrm{ax}+\mathrm{b})^{2}$, where $a$ and $b$ are integers to be found.


Figure 2
Figure 2 shows a sketch of part of the curve with equation $y=g(x)$.
(c) Use your answer to part (b), and the sketch, to deduce the values of $x$ for which
(i) $\quad g(x) \leq 0$
(ii) $g(2 x)=0$
(Total for question = 9 marks)

Q3.

$$
f(x)=2 x^{3}-13 x^{2}+8 x+48
$$

(a) Prove that $(x-4)$ is a factor of $\mathrm{f}(x)$.
(b) Hence, using algebra, show that the equation $\mathrm{f}(x)=0$ has only two distinct roots.
(4)


Figure 2
Figure 2 shows a sketch of part of the curve with equation $y=\mathrm{f}(x)$.
(c) Deduce, giving reasons for your answer, the number of real roots of the equation

$$
\begin{equation*}
2 x^{3}-13 x^{2}+8 x+48=0 \tag{2}
\end{equation*}
$$

Given that $k$ is a constant and the curve with equation $y=\mathrm{f}(x+k)$ passes through the origin,
(d) find the two possible values of $k$.

## Q4.

$$
g(x)=2 x^{3}+4 x^{2}-41 x-70
$$

(a) Use the factor theorem to show that $\mathrm{g}(x)$ is divisible by $(x-5)$.
(b) Hence, showing all your working, write $\mathrm{g}(x)$ as a product of three linear factors.
(4)

The finite region $R$ is bounded by the curve with equation $y=\mathrm{g}(x)$ and the $x$-axis, and lies below the $x$-axis.
(c) Find, using algebraic integration, the exact value of the area of $R$.

Q5.

$$
\mathrm{f}(\mathrm{x})=2 x^{3}-5 x^{2}+a x+a
$$

Given that $(x+2)$ is a factor of $\mathrm{f}(x)$, find the value of the constant $a$.
(3)
(Total for question = 3 marks)

Q6.

$$
\mathrm{f}(x)=-3 x^{3}+8 x^{2}-9 x+10, \quad x \in \mathbb{R}
$$

(a) (i) Calculate f (2)
(ii) Write $\mathrm{f}(x)$ as a product of two algebraic factors.
(3)

Write $\mathrm{f}(x)$ as a product of two algebraic factors.
(b) prove that there are exactly two real solutions to the equation

$$
\begin{equation*}
-3 y^{6}+8 y^{4}-9 y^{2}+10=0 \tag{2}
\end{equation*}
$$

(c) deduce the number of real solutions, for $7 \pi \leq \theta<10 \pi$, to the equation

$$
3 \tan ^{3} \theta-8 \tan ^{2} \theta+9 \tan \theta-10=0
$$

(Total for question = 6 marks)

Q7.

$$
f(x)=3 x^{3}+2 a x^{2}-4 x+5 a
$$

Given that $(x+3)$ is a factor of $\mathrm{f}(x)$, find the value of the constant $a$.
(Total for question = 3 marks)

Q8.

$$
f(x)=a x^{3}+10 x^{2}-3 a x-4
$$

Given that $(x-1)$ is a factor of $\mathrm{f}(x)$, find the value of the constant $a$. You must make your method clear.


- Founder \& CEO of Chemistry Online Tuition Ltd.
- Tutoring students in UK and worldwide since 2008
- CIE \& EDEXCEL Examiner since 2015
- Chemistry, Physics, and Math's Tutor


## CONTACT INFORMATION FOR

CHEMISTRY ONLINE TUITION

- UK Contact: 02081445350
- International Phone/WhatsApp: 00442081445350
- Website: www.chemistryonlinetuition.com
- Email: asherrana@chemistryonlinetuition.com

Address: 210-Old Brompton Road, London SW5 OBS, UK

