



CHEMISTRY ONLINE
— TUITION —

Phone: +442081445350

www.chemistryonlinetuition.com

Email:asherrana@chemistryonlinetuition.com

PURE MATH

ALGEBRA AND FUNCTION

Level & Board	EDEXCEL (A-LEVEL)
TOPIC:	INTEGRATION
PAPER TYPE:	SOLUTION - 9
TOTAL QUESTIONS	8
TOTAL MARKS	26

INTEGRATION - 9

1. Let $y = \sqrt[4]{x} = x^{1/4}$

we take $x = 16$

$$\delta x = dx = 17 - 16 = 1$$

$$y = (16)^{1/4} = 2$$

$$\text{Now } y = x^{1/4}$$

$$d(y) = d(x^{1/4})$$

$$d(y) = \frac{1}{4} x^{\frac{1}{4}-1} dx$$

$$d(y) = \frac{1}{4} x^{-\frac{3}{4}} dx$$

$$\text{Put } x = 16, dx = 1$$

$$dy = \frac{1}{4} (16)^{-\frac{3}{4}} (1) = \frac{1}{4} (2)^{-3}$$

$$dy = \frac{1}{4} \cdot \frac{1}{8} = \frac{1}{32} = 0.03125$$

$$\text{Thus } \sqrt[4]{17} \approx y + dy$$

$$\sqrt[4]{17} = 2 + 0.03125$$

$$\sqrt[4]{17} = 2.03125$$

2. $y = x^2 - 1$ As x changes from 3 to 3.02, so

$$y = x^2 - 1$$

$$d(y) = d(x^2 - 1)$$

$$dy = 2x dx - 0 = 2x dx$$

Put the value of x and dx

$$dy = 2(3)(0.02) = 0.12$$

Now

$$y + \delta y = (x + \delta x)^2 - 1$$

$$\delta y = (x + \delta x)^2 - 1 - y$$

Put value of y

$$\delta y = (x + \delta x)^2 - 1 - (x^2 - 1)$$

$$\delta y = (x + \delta x)^2 - 1 - x^2 + 1$$

$$\delta y = (x + \delta x)^2 - 1 - x^2$$

$$x = 3$$

$$\delta x = dx = 3.02 - 3 = 0.02$$

Put the value of c and δx

$$\delta y = (3 + 0.02)^2 - (3)^2$$

$$\delta y = 0.1204$$

3. $y = x^2 + 2x$

Now

$$y = x^2 + 2x$$

$$d(y) = d(x^2 + 2x)$$

$$dy = 2xdx + 2dx$$

Put the value of x and dx

$$dy = 2(2)(-0.2) + 2(-0.2)$$

$$dy = -1.2$$

Now

$$y + \delta y = (x + \delta x)^2 + 2(x + \delta x)$$

$$\delta y = (x + \delta x)^2 + 2x + 2\delta x - y$$

Put the value of y

$$\delta y = (x + \delta x)^2 + 2x + 2\delta x - (x^2 + 2x)$$

$$\delta y = (x + \delta x)^2 + 2x + 2\delta x - x^2 - 2x$$

$$\delta y = (x + \delta x)^2 + 2\delta x - x^2$$

$$x = 2$$

$$\delta y = (2 - 0.2)^2 + 2(-0.2) - (2)^2$$

$$\delta y = -1.16$$

4. $= \int (\sqrt{x} - \frac{1}{\sqrt{x}}) dx$

$$= \int \sqrt{x} dx + \int \frac{1}{\sqrt{x}} dx$$

$$= \int x^{1/2} dx + \int x^{-1/2} dx$$

I am Sorry !!!!!

$$\begin{aligned}
 &= \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + \frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + c \\
 &= \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c \\
 &= \frac{2}{3}x^{3/2} + 2x^{1/2} + c
 \end{aligned}$$

5. $= \int (2x + 3)^{1/2} dx$
 × and ÷ by 2 to make a derivative

$$\begin{aligned}
 &= \frac{1}{2} \int (2x + 3)^{1/2} \cdot 2 dx \\
 &= \frac{1}{2} \cdot \frac{(2x+3)^{1/2+1}}{1/2+1} + c \\
 &= \frac{1}{2} \cdot \frac{(2x+3)^{3/2}}{3/2} + c \\
 &= \frac{1}{2} \cdot \frac{2}{3} (2x + 3)^{3/2} + c \\
 &= \frac{1}{3} (2x + 3)^{3/2} + c
 \end{aligned}$$

6. $xy + x = 4$

Taking differentials on both sides

$$d(xy + x) = d(4)$$

$$xdy + ydx + dx = 0$$

$$xdy + (y + 1)dx = 0$$

$$xdy = -(y + 1)dx$$

$$\frac{dy}{dx} = -\frac{y+1}{x} \quad \text{and} \quad \frac{dx}{dy} = -\frac{x}{y+1}$$

$$7. \quad = \int \frac{e^{2x} + e^x}{e^x} dx$$

$$= \int \left[\frac{e^{2x}}{e^x} + \frac{e^x}{e^x} \right] dx$$

$$= \int [e^x + 1] dx$$

$$= \int e^x dx + \int 1 dx$$

$$= \frac{e^x}{1} + x + c$$

$$= e^x + x + c$$

$$8. \quad = \int \tan^2 x dx$$

$$= \int (\sec^2 x - 1) dx$$

$$\therefore 1 + \tan^2 \theta = \sec^2 \theta$$

$$= \int \sec^2 x dx - \int 1 dx$$

$$= \tan x - x + c$$

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DR. ASHAR RANA



Phone: +442081445350
www.chemistryonlinetuition.com
Email: asherrana@chemistryonlinetuition.com

- Founder & CEO of Chemistry Online Tuition Ltd.
- Tutoring students in UK and worldwide since 2008
- Chemistry, Physics, and Math's Tutor

CONTACT INFORMATION FOR **CHEMISTRY ONLINE TUITION**

- UK Contact: 02081445350
- International Phone/WhatsApp: 00442081445350
- Website: www.chemistryonlinetuition.com
- Email: asherrana@chemistryonlinetuition.com
- Address: 210-Old Brompton Road, London SW5 0BS, UK