



**CHEMISTRY ONLINE**  
— **TUITION** —

Phone: +442081445350

[www.chemistryonlinetuition.com](http://www.chemistryonlinetuition.com)

Email: [asherrana@chemistryonlinetuition.com](mailto:asherrana@chemistryonlinetuition.com)

# **PURE MATH**

## **ALGEBRA AND FUNCTION**

|                 |                   |
|-----------------|-------------------|
| Level & Board   | EDEXCEL (A-LEVEL) |
| TOPIC:          | INTEGRATION       |
| PAPER TYPE:     | SOLUTION - 1      |
| TOTAL QUESTIONS | 8                 |
| TOTAL MARKS     | 26                |

**ChemistryOnlineTuition Ltd reserves the right to take legal action against any individual/ company/organization involved in copyright abuse.**

**INTEGRATION – 1**

$$\begin{aligned}
 1. \quad & \frac{y}{x} - \ln x = \ln c \\
 \Rightarrow & d\left(\frac{y}{x} - \ln x\right) = d(\ln c) \\
 & d\left(\frac{y}{x}\right) - d(\ln x) = 0 \\
 \Rightarrow & \left(\frac{xdy - ydx}{x^2}\right) - \frac{1}{x}dx = 0 \\
 & \left(\frac{xdy - ydx}{x^2}\right) = \frac{1}{x}dx \\
 \Rightarrow & xdy - ydx = \frac{1}{x}dx \\
 & xdy - ydx = xdx \\
 & xdy = ydx + xdx \\
 \Rightarrow & xdy = (y + x)dx \\
 & \frac{dy}{dx} = \frac{x + y}{x}
 \end{aligned}$$

$$\begin{aligned}
 2. \quad & \text{Let } y = \sqrt[4]{x} = x^{1/4} \\
 & \text{we take } x = 16 \\
 & \delta x = dx = 17 - 16 = 1 \\
 & y = (16)^{1/4} = 2 \\
 & \text{Now } y = x^{1/4} \\
 & d(y) = d(x^{1/4}) \\
 & d(y) = \frac{1}{4}x^{\frac{1}{4}-1}dx \\
 & d(y) = \frac{1}{4}x^{-\frac{3}{4}}dx \\
 & \text{Put } x = 16, dx = 1 \\
 & dy = \frac{1}{4}(16)^{-\frac{3}{4}}(1) = \frac{1}{4}(2)^{-3} \\
 & dy = \frac{1}{4} \cdot \frac{1}{8} = \frac{1}{32} = 0.03125 \\
 & \text{Thus } \sqrt[4]{17} \approx y + dy
 \end{aligned}$$

$$\sqrt[4]{17} = 2 + 0.03125$$

$$\sqrt[4]{17} = 2.03125$$

3. Let the radius of a circular disc =  $x$  cm

$$\text{Area of a disc} = \pi r^2$$

$$A = \pi r^2$$

$$d(A) = d(\pi r^2)$$

$$dA = \pi \cdot 2x dx$$

As the diameter changes from 44 to 44.4,

So the radius changes from 22 to 22.4, so

$$x = 22, dx = 22.2 - 22 = 0.2$$

$$dA = \pi(2)(22)(0.2)$$

$$dA = 27.646 \text{ cm}^2$$

4. 
$$= \int 3x^2 dx - \int 2x dx + \int 1 dx$$

$$= 3 \int x^2 dx - 2 \int x dx + \int 1 dx$$

$$= 3 \cdot \frac{x^{2+1}}{2+1} - 2 \cdot \frac{x^{1+1}}{1+1} + x + c$$

$$= 3 \cdot \frac{x^3}{3} - 2 \cdot \frac{x^2}{2} + x + c$$

$$= x^3 - x^2 + x + c$$

5.  $= \int (2x + 3)^{1/2} dx$   
 $\times$  and  $\div$  by 2 to make derivative  
 $= \frac{1}{2} \int (2x + 3)^{1/2} \cdot 2 dx$   
 $= \frac{1}{2} \cdot \frac{(2x+3)^{1/2+1}}{1/2+1} + c$   
 $= \frac{1}{2} \cdot \frac{(2x+3)^{3/2}}{3/2} + c$   
 $= \frac{1}{2} \cdot \frac{2}{3} (2x + 3)^{3/2} + c$   
 $= \frac{1}{3} (2x + 3)^{3/2} + c$

6. Length of each edge of a cube =  $x$  unit

Volume of cube =  $L.W.H$

$$V = x \cdot x \cdot x$$

$$d(V) = (x^3)$$

$$dV = 3x^2 dx$$

When  $x$  changes from 5 to 5.02, so

$$x = 5, dx = 5.02 - 5 = 0.02$$

$$dV = 3(5)^2(0.02) = 1.5 \text{ cube units}$$

$$\begin{aligned}
 7. \quad &= \int \frac{1}{(2x+3)^4} dx \\
 &= \int (2x+3)^{-4} dx \\
 &= \frac{1}{2} \cdot \frac{(2x+3)^{-4+1}}{-4+1} + c \\
 &= -\frac{1}{6(2x+3)^3} + c
 \end{aligned}$$

8. We know that

$$\frac{d}{dx} [\ln f(x)] = \frac{1}{f(x)} \cdot f'(x)$$

Taking integration on both sides

$$\int \frac{d}{dx} [\ln f(x)] = \int \frac{1}{f(x)} \cdot f'(x)$$

$$\Rightarrow \ln f(x) = \int \frac{f'(x)}{f(x)} dx$$

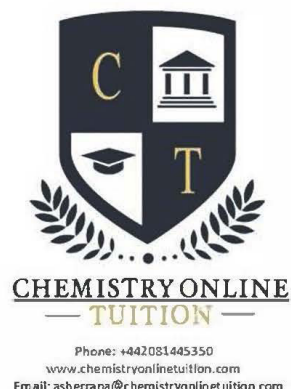
$$\Rightarrow \int \frac{f'(x)}{f(x)} dx = \ln f(x) + c \quad \text{By definition}$$

$$(\int f(x) dx = F(x) + c)$$

Hence proved.



**DR. ASHAR RANA**



- Founder & CEO of Chemistry Online Tuition Ltd.
- Tutoring students in UK and worldwide since 2008
- Chemistry, Physics, and Math's Tutor

---

## CONTACT INFORMATION FOR CHEMISTRY ONLINE TUITION

- UK Contact: 02081445350
- International Phone/WhatsApp: 00442081445350
- Website: [www.chemistryonlinetuition.com](http://www.chemistryonlinetuition.com)
- Email: [asherrana@chemistryonlinetuition.com](mailto:asherrana@chemistryonlinetuition.com)
- Address: 210-Old Brompton Road, London SW5 OBS, UK