PURE MATH

ALGEBRA AND FUNCTION

Level & Board	EDEXCEL (A-LEVEL)
TOPIC:	CIRCLES
PAPER TYPE:	SOLUTION - 3
TOTAL QUESTIONS	8
TOTAL MARKS	64

Q.1

(a)

(i) Using the general form of the equation of a circle is:

$(x - h)^2 + (y - k)^2 = r^2 \rightarrow (i)$

Using given equation of the circle

$x^2 + y^2 - 4x + 8y - 8 = 0$

$\Longrightarrow$

$(x^2 - 4x) + (y^2 + 8y) - 8 = 0$

$\Longrightarrow$

$(x^2 - 4x + 4) + (y^2 + 8y + 16) - 8 - 4 - 16 = 0$

$\Longrightarrow$

$(x - 2)^2 + (y + 4)^2 - 28 = 0 \rightarrow (ii)$

So, the center of the circle (h,k) is (2,-4)

(ii) The radius r cab be found from the equation $(x - h)^2 + (y - k)^2 = r^2$

$\Longrightarrow \quad r^2 = 28$

$\Longrightarrow \quad \sqrt{r^2} = \sqrt{28}$

$\Longrightarrow \quad r = \sqrt{4 \times 7}$

$\Longrightarrow \quad r = 2\sqrt{7}$

(b) The line r = k is a vertical line for this line to be tangent to the circle, the distance between the center of the center and the line should be equal to the radius of the circle. The distance between a point (a , b) and the line z = k is given by $|a - k|$ So, for the tangent condition.

$|2 - k| = 2\sqrt{7}$

$\Longrightarrow \quad 2 - k = \pm 2\sqrt{7}$

$\Longrightarrow \quad 2 - k = 2\sqrt{7} \quad or \ 2 - k = -2\sqrt{7}$

$\Longrightarrow \quad k = 2 - 2\sqrt{7} \quad or \ \ k = 2 + 2\sqrt{7}$

So, the range of values for k is $2 - 2\sqrt{7} \leq k \leq 2 + 2\sqrt{7}$

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Q.2

(a)

(i) We know that $(x-h)^2 + (y-k)^2 = r^2 \rightarrow (i)$

Now, given:

$\implies \quad x^2 + y^2 - 6x - 4y + 12 = 0$

$\implies \quad (x^2 - 6x + 9) + (y^2 - 4y + 4) + 12 - 9 - 4 = 0$

$\implies \quad (x-3)^2 + (y-2)^2 + 3 = 0 \rightarrow (ii)$

Comparing (i) and (ii), we get

The center of the circle is (3,2)

(ii) The radius r can be found from the equation $(x-h)^2 + (y-k)^2 = r^2$

In our case,

$r^2 = 3$

So,

$r = \sqrt{3}$

(c) For the line x = k to be the circle

$\implies \quad |3 - k| = \sqrt{3}$

$\implies \quad 3 - k = \pm\sqrt{3}$

$\implies \quad 3 - k = \sqrt{3} \quad or \quad 3 - k = -\sqrt{3}$

$\implies \quad k = 3 - \sqrt{3} \quad or \quad k = 3 + \sqrt{3}$

So, the range of values for k is $3 - \sqrt{3} \le k \le 3 + \sqrt{3}$

Q.3

(a)

To find the Centre, complete the square for both x and y

$\implies \quad x^2 - 8x + y^2 + 6y = -16$

$\implies \quad (x^2 - 8x + 16) + (y^2 + 6y + 9) = -16 + 16 + 9$

$\implies \quad (x-4)^2 + (y+3)^2 = 9 \rightarrow (i)$

$\therefore \quad$ Using standard form of the circle

$\implies \quad (x-h)^2 + (y-k)^2 = r^2 \ (ii)$

$\implies \quad$ Centre (h , k) = (4 , -3)

(b) we are find the value of radius.

Comparing (i) and (ii) we get

$$\implies \quad r = \sqrt{9} \implies r = 3$$

So, the radius of S is 3

Q.4

(a)

(i) Coordinates of the Centre of C

$$x^2 + y^2 + 2x - 6y + 5 = 0$$

$$x^2 + 2x + y^2 - 6x + 5 = 0$$

$$(x^2 + 2x + 1) + (y^2 - 6y + 9) + 5 - 1 - 9 = 0$$

$$(x + 1)^2 + (y - 3)^2 + 4 = 0$$

So,

The Centre of the circle is (-1 , 3)

(ii) The radius r can be found from the equation $(x - h)^2 + (y - k)^2 = r^2$

In our case

$$r^2 = 4$$

So, $\quad r = 2$

(b) for the line $\therefore x = k$

$$|-1 - k| = 2$$

$$\implies \quad -1 - k = \pm 2$$

$$\implies \quad -1 - k = 2 \quad or \quad -1 - k = -2$$

$$\implies \quad k = -3 \qquad or \quad k = 1$$

S, the range of values for k is $-3 \leq k \leq 1$

I am Sorry !!!!!

Q.5

(a)

Given that circle C with center (-2,6) passes through the point (10,11)

Let's denoted the radius of the circle as r.

We know that

$$(x - h)^2 + (y - k)^2 = r^2$$

$$(x + 2)^2 + (y - 6)^2 = r^2$$

Now,

$$p(10,11) = p(x, y)$$

$$(10 + 2)^2 + (11 - 6)^2 = r^2$$

$$\implies r^2 = 144 + 25$$

$$\implies r^2 = 169$$

So, the circle C indeed passes through the point (10,1)

(b)

Let's denote the points P and Q as the point where the tangent lines at (10,11) and (10,1) respectively, intersect the y-axis.
Let m_1 and m_2 be the slope of the tangent at (10,11) and (10,1)
Now, The equation of the tangent in point − slope form are:

$$\implies y - y_1 = m_1(x - x_1)$$

$$\implies y - 11 = m_1(x - 10)$$

$$\implies y - y_1 = m_2(x - 10)$$

To find the y − intercepts (P and Q)

For P: $y_P = 11 - 10m_1$

For Q: $y_Q = 1 - 10m_2$

Using distance formula

$$|pq| = d = \sqrt{(0 - 0)^2 + (y_Q - y_P)^2}$$

Now, Substitute the values into the formula for PQ and simplify.

If done correctly, you should obtain PQ = 58

Q.6

The distance between two points x_1, y_1 and x_2, y_2 in a plane is given by the

Distance formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = |PQ| = \sqrt{(8 - 3)^2 + (4 - 2)^2}$$

$$PQ = \sqrt{25 + 4}$$

$$PQ = \sqrt{29}$$

Q.7

(a) The equation of the circle is given as:

$$x^2 + y^2 - 6x + 10y + 9 = 0$$

To write this equation in the standard form

$$(x - h)^2 + (y - k)^2 = r^2 \quad \rightarrow (i)$$

Now,

$$x^2 - 6x + y^2 + 10y = -9$$

$$(x^2 - 6x + 9) + (y^2 + 10y + 25) = -9 + 9 + 25$$

$$(x - 3)^2 + (y + 5)^2 = 25 \quad \rightarrow (ii)$$

Now,

Comparing the equation (i) and (ii)

$$\implies \quad Center \ (h, k) = (3, 5)$$

$$\implies \quad radius \ (r) = 5$$

So, the center of the circle is (3,-5) and the radius is 5.

(b) The line y = Kx intersect the circle at two distinct points.
Substituting this into the circle equation, we get

$$x^2 + (kx)^2 - 6x + 10(kx) + 9 = 0$$

$$x^2(1 + k^2) - 6x + 10kx + 9 = 0$$

Here,

$$a = 1 + k^2, b = 4(2.5)k, c = 9$$

Now,

$$b^2 - 4ac \geq 0$$

The discriminant must be greater than or equal to zero

$$\left(4(2.5k)\right)^2 - 4(1 + k^2)(a) \geq 0$$

$$100k^2 - 36 - 36k^2 \geq 0$$

$$64k^2 \geq 36$$

$$k^2 \geq \frac{9}{16}$$

$$k \geq \frac{3}{4} \ or \ k \leq \frac{-3}{4}$$

So, the range of values for k is

$$k \geq \frac{3}{4} \ or \ k \leq \frac{-3}{4}$$

Q.8

(a) *Given equation of circle*

$$x^2 - 8x + y^2 + 6y = -16$$

$$(x^2 - 8x + 16) + (y^2 + 6y + 9) = -16 + 16 + 9$$

$$(x - 4)^2 + (y + 3)^2 = 9$$

$$(x - 4)^2 + (y + 3)^2 = (3)^2 \ \rightarrow (i)$$

Using standard form of circle

$$(x - h)^2 + (y - k)^2 = (r)^2 \rightarrow (ii)$$

Comparing (i) and (ii) , we get

$$Center \ (h, k) = (4, -3)$$

(b) $radius \ (r) = 3$

(c) *Consider the line y = kx , where k is a constant*

Substituting y = kx into the circle equation

$$x^2 + (kx)^2 - 2x + 4(kx) + 5 = 0$$

$$x^2(1 + k^2) + 2x(2k) + 5 = 0$$

For real solution , the discriminant must be greater than or equal to zero

$$\left(2(2k)\right)^2 - 4(1 + k^2)(5) \geq 0$$

Simplify and solve for k:

$$16k^2 - 20(1 + k^2) \geq 0$$

$$16k^2 - 20 - 20k^2 \geq 0$$

$$-4k^2 - 20 \geq 0$$

$$k^2 + 5 \leq 0$$

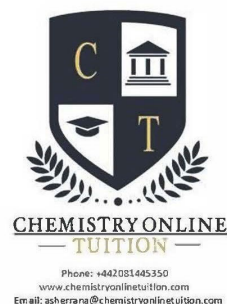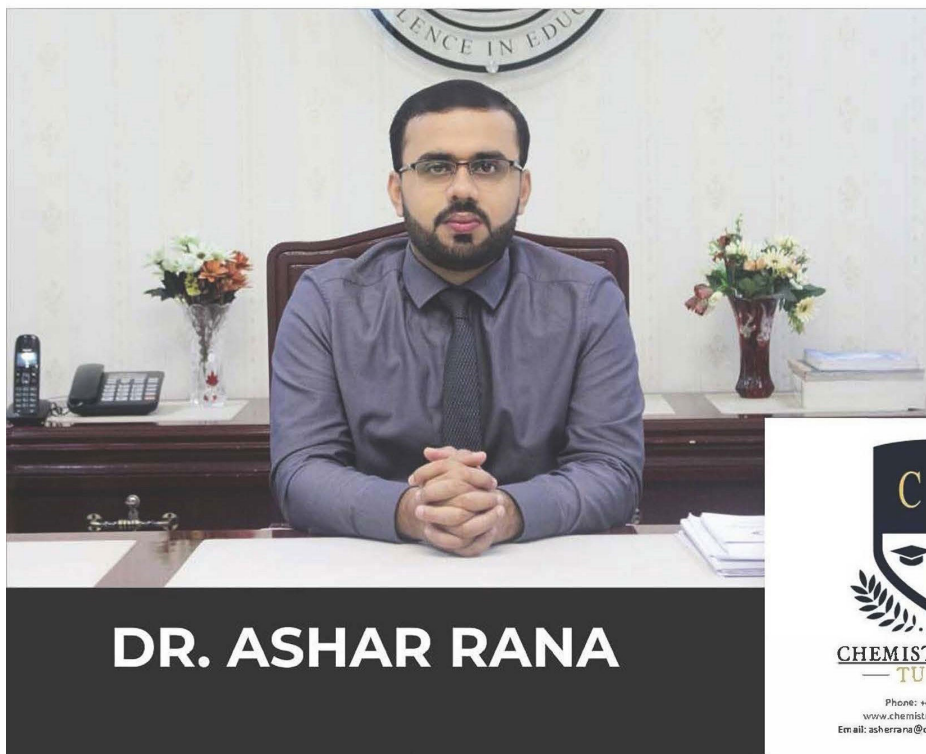This inequality has no real solution for k, meaning that any line y = kx where

K is a constant will not intersect the circle C_4 , except the ease when the line Coincides with the circle.

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