

## CHEMISTRY ONLINE

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Phone: +442081445350
www.chemistryonlinetuition.com

Emil:asherrana@chemistryonlinetuition.com
PURE MATH
ALGEBRA AND FUNCTION

Level \& Board
EDEXCEL (A-LEVEL)

TOPIC: CIRCLES

PAPER TYPE:
SOLUTION - 1

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(a) Given that circle $C$ with center $(-2,6)$ passes through the point $(10,11)$ Let's denoted the radius of the circle as $r$.
We know that
$(x-h)^{2}+(y-k)^{2}=r^{2}$
$(x+2)^{2}+(y-6)^{2}=r^{2}$
Now,
$p(10,11)=p(x, y)$
$(10+2)^{2}+(11-6)^{2}=r^{2}$
$\Rightarrow \quad r^{2}=144+25$
$\Rightarrow \quad r^{2}=169$
So, the circle C indeed passes through the point $(10,1)$
(b) Let's denote the points P and Q as the point where the tangent lines at $(10,11)$ and $(10,1)$ respectively, intersect the $y$-axis .
Let $m_{1}$ and $m_{2}$ be the slope of the tangent at $(10,11)$ and $(10,1)$
Now, The equation of the tangent in point - slope form are:

$$
\begin{array}{ll}
\Rightarrow & y-y_{1}=m_{1}\left(x-x_{1}\right) \\
\Rightarrow & y-11=m_{1}(x-10) \\
\Rightarrow & y-y_{1}=m_{2}(x-10)
\end{array}
$$

To find the y - intercepts ( P and Q )
For P: $y_{P}=11-10 m_{1}$
For $\mathrm{Q}: y_{Q}=1-10 m_{2}$
Using distance formula
$|p q|=d=\sqrt{(0-0)^{2}+\left(y_{Q}-y_{P}\right)^{2}}$
Now, Substitute the values into the formula for PQ and simplify.
If done correctly, you should obtain $P Q=58$
Q. 2
(a) Using distance formula

$$
\begin{aligned}
& d=\sqrt{\left(x_{2}-x_{1}\right)^{2}+\left(y_{2}-y_{1}\right)^{2}} \\
& d=\sqrt{\left((-3)^{2}-5\right)^{2}+(4-(-2))^{2}} \\
& d=\sqrt{64+36} \\
& d=\sqrt{100} \\
& d=10
\end{aligned}
$$

Thus, the distance of $x y$ is 10
(b) Using mid-point formula

$$
\begin{aligned}
& m=\left(\frac{x_{2}-x_{1}}{2}, \frac{y_{2}-y_{1}}{2}\right) \\
& m=\left(\frac{-3+5}{2}, \frac{4+(-2)}{2}\right) \\
& m=(1,1)
\end{aligned}
$$

The slope of the hypoteneouse $x y$ is given by:

$$
\begin{aligned}
& m_{x y}=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}=\frac{-2-4}{5+3} \\
& m_{x y}=\frac{-3}{4}
\end{aligned}
$$

$\Rightarrow \quad$ The slope of the perpendicular bisector is $\frac{4}{3}$
$\Rightarrow \quad y-y_{1}=m\left(x-x_{1}\right)$
$y-1=\frac{4}{3}(x-1)$
$3(y-1)=4(x-1)$
$3 y-3=4 x-4$
$3 y=4 x-4+3$
$3 y=4 x-1$

This is the equation of the perpendicular bisector.
(c) The perpendicular bisector has the equation $3 y=4 x-1$

$$
\begin{aligned}
& \text { Put } m(1,1) \text { of } x y \\
\Rightarrow & 3(1)=4(1)-1 \\
\Rightarrow & 3=4-1 \\
\Rightarrow & 3=3
\end{aligned}
$$

The coordinates of the circumcenter are $m(1,1)$
Q. 3

Solution:
We know that
The slope ( m ) of a line passing through two points $\left(x_{1}, y_{1}\right)$ and $\left(x_{2}, y_{2}\right)$
Given by:

$$
\begin{aligned}
& \Rightarrow \quad \frac{y_{2}-y_{1}}{x_{2}-x_{1}} \\
& \Rightarrow \quad m_{a b}=\frac{8-3}{7-2}=\frac{5}{5}=1 \\
& \Rightarrow \quad m_{c d}=\frac{4-(-1)}{-1-4}=\frac{5}{5}=1
\end{aligned}
$$

Since, $\quad m_{a b}=m_{c d}$
The opposite sides $A B$ and $C D$ are parallel
Q. 4

## Solution

The distance between two points $x_{1}, y_{1}$ and $x_{2}, y_{2}$ in a plane is given by the Distance formula

$$
\begin{aligned}
& d=\sqrt{\left(x_{2}-x_{1}\right)^{2}+\left(y_{2}-y_{1}\right)^{2}} \\
& d=|P Q|=\sqrt{(8-3)^{2}+(4-2)^{2}} \\
& P Q=\sqrt{25+4} \\
& P Q=\sqrt{29}
\end{aligned}
$$

Q. 5

## Solution:

The mid - point $m$ of $Q R$ is given by:
$M=\left(\frac{x_{Q}+x_{R}}{2}, \frac{y_{Q}+y_{R}}{2}\right)$
Similarly,
find the midpoints $N$ and $O$ for $R P$ and $P Q$ respectively.

The equation of the line passing through two points
$\left(x_{1}, y_{1}\right)$ and $\left(x_{2}, y_{2}\right)$ is given by
$y-y_{1}=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}\left(x-x_{1}\right)$

For Pm, QN and RO

Substitute the mid - point coordinates to find the equation
Q. 6

## Solution:

The distance (d) from a point $\left(x_{0}, y_{0}\right)$ to a line $A x+B y+c=0$
Is given by the formula,
$d=\frac{\left|A x_{0}+B y_{0}+c\right|}{\sqrt{A^{2}+B^{2}}}$
For this line $\mathrm{AC}: ~ y=-2 x+7$ and the point $B(7,8)$
$\Rightarrow \quad d=\frac{|-2(7)+1(8)-7|}{\sqrt{(-2)^{2}+(1)^{2}}}$
$\Rightarrow \quad d=\frac{|-14+8-7|}{\sqrt{4+1}}$
$\Rightarrow \quad d=\frac{|-13|}{\sqrt{5}}$
$\therefore$ Rationalize
$\Rightarrow \quad d=\frac{13}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$
$\Rightarrow \quad d=\frac{13 \sqrt{5}}{5}$
Q. 7

Solution:
(a) Using the equation of circle
$\Rightarrow \quad(x-h)^{2}+(y-k)^{2}=r^{2}$
$\Rightarrow \quad(x+3)^{2}+(y-5)^{2}=r^{2}$
(b) Since point $A(4,2)$ lines on the circle, we can substitute these coordinates Into the equation of the circle and solve for $r$ :

$$
\begin{aligned}
& (4+3)^{2}+(2-5)^{2}=r^{2} \\
& (7)^{2}+(-3)^{2}=r^{2} \\
& 49+9=r^{2} \\
& r^{2}=58 \\
& r=\sqrt{58}
\end{aligned}
$$

Q. 8

## Solution:

The equation of a line passing through two points $\left(x_{1}, y_{1}\right)$ and $\left(x_{2}, y_{2}\right)$ is given by:

$$
\begin{aligned}
& \Rightarrow \quad y-y_{1}=m\left(x-x_{1}\right) \\
& \Rightarrow \quad y-y_{1}=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}\left(x-x_{1}\right) \\
& \Rightarrow \quad y-3=\frac{(-1-3)}{(4-2)}(x-2) \\
& \Rightarrow \quad y-3=\frac{-4}{2}(x-2) \\
& \Rightarrow \quad y-3=-2(x-2) \\
& \Rightarrow \quad y-3=-2 x+4 \\
& \Rightarrow \quad y=-2 x+4+3 \\
& \Rightarrow \quad y=-2 x+7
\end{aligned}
$$

So, the equation of the line containing the diagonal A is $y=2 x+7$


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## CONTACT INFORMATION FOR

## CHEMISTRY ONLINE TUITION

- UK Contact: 02081445350
- International Phone/WhatsApp: 00442081445350
- Website: www.chemistryonlinetuition.com
- Email: asherrana@chemistryonlinetuition.com

Address: 210-Old Brompton Road, London SW5 OBS, UK

