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PURE MATH

ALGEBRA AND FUNCTION

Level & Board	EDEXCEL (A-LEVEL)
TOPIC:	CIRCLES
PAPER TYPE:	SOLUTION - 4
TOTAL QUESTIONS	8
TOTAL MARKS	64

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Q.1

(i) The general form of a circle equation

$$x^2 + y^2 + 2ax + 2by + c = 0 \ \rightarrow (ii)$$

Given equation

$$x^2 + y^2 + 18x - 2y + 30 = 0 \ \rightarrow (ii)$$

Comparing equation (i) and (ii)

$$2a = 18 \ \rightarrow \ a = 9$$

$$2b = -2 \ \rightarrow \ b = -1$$

$$c = 30$$

So, the Centre of circle C is (-9,1)

The given point P(-5,7)

$\Longrightarrow$

$$r = \sqrt{(x_P - a)^2 + (y_P - b)^2}$$

$$r = \sqrt{(-5+9)^2 + (7-1)^2}$$

$$r = \sqrt{16 + 36}$$

$$r = \sqrt{52}$$

$$r = \sqrt{4 \times 13}$$

$$r = 2\sqrt{13}$$

Now, The equation of the tangent at the point (-5,7) is given by:

$$x(x_P) + y(y_P) = r^2$$

$$(x+5) + (y-7) = \left(2\sqrt{13}\right)^2$$

$$x + y - 2 = 4(13)$$

$$x + y = 52 - 2$$

$$x + y - 50 = 0$$

So, the required equation of the tangent line is x + y – 50 = 0

(ii) where,

$$a = \frac{-8}{2} = -4 \ and \ b = \frac{12}{2} = 6$$

$\Longrightarrow$ the Centre is $(-9\,,6)$

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So,

$$\sqrt{(-4)^2 + (6)^2 - k} < 4$$

$\Longrightarrow$

$$16 + 36 - k < 4$$

$$52 - k < 16$$

$$-k < -36$$

$$k > 36$$

Therefore,

The range of possible values for k is $k > 36$

Q.2

As circle lies in 2nd quadrant with radius 'a' and tangent to both axes, therefore

Centre: (-a ,a)

Required equation of circle is

$$(x - h)^2 + (y - k)^2 = r^2$$

$$(x + a)^2 + (y - a)^2 = r^2$$

$$x^2 + a^2 + 2ax + y^2 + a^2 - 2ay - a^2 = 0$$

$$x^2 + y^2 + 2ax - 2ay + a^2 = 0$$

$$(f + 13)(f + 5) = 0$$

$$Either\ f + 13 = 0$$

$$f = -13$$

OR

$$f + 5 = 0$$

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$f = -5$

$When\ f = -13$

$Eq(3) \implies g - 2(-13) - 12 = 0$

$g + 26 - 12 = 0 \implies g = -14$

$Eq(2) \implies -2(-14) + 16(-13) + c + 65 = 0$

$\implies 28 - 208 + c + 65 = 0$

$\implies c - 115 = 0 \implies c = 115$

Putting values in eq(i)

$x^2 + y^2 + 2(-14)x + 2(-13)y + 115 = 0$

$x^2 + y^2 - 28x - 26y - 115 = 0$

When f = -5

$Eq(3) \implies g - 2(-5) - 12 = 0$

$g + 10 - 12 = 0 \implies g = 2$

Eq(2) $\implies -2(2) + 16(-5) + c + 65 = 0$

$-4 - 80 + c = 65 = 0$

$\implies c = 19$

Putting values in eq (1)

$x^2 + y^2 + 2(2)x + 2(-5)y + 19 = 0$

$x^2 + y^2 + 4x - 10y + 19 = 0$

Q.3

$l_1 \quad 3x - 2y = 0$

$l_2 \quad 2x + 3y - 13 = 0$

$Circle \quad x^2 + y^2 + 6x - 4y = 0$

Comparing it with

$x^2 + y^2 + 2gx + 2fy + c = 0$

$\implies 2g = 6\,, 2f = -4\,, c = 0$

$g = 3\,, f = -2$

$Centre := (-g\,, -f) = (-3\,, 2)$

$Radius = \sqrt{g^2 + f^2 - c} = \sqrt{13}$

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$$= \sqrt{(3)^2 + (-2)^2 - 0} = \sqrt{9+4}$$

$Distacne\ from\ centre(-3\ ,2)\ to\ line\ l_1 = \frac{|3(-3)-2(2)|}{\sqrt{(3)^2+(-2)^2}}$

$= \frac{|13|}{\sqrt{13}} = \sqrt{13} = r \quad \ldots\ldots(1)$

$Distance\ from\ centre(-3\ ,2)\ to\ line\ l_2$

$= \frac{|2(-3)+3(2)-13|}{\sqrt{(2)^2+(3)^2}}$

$= \frac{|13|}{\sqrt{13}} = \sqrt{13} = r \quad \ldots\ldots(2)$

$From\ (1) and\ (2) it\ is\ clear\ that\ l_1\ and\ l_2 are\ tangent\ to\ circle$

`Q.4

$S_1: x^2 + y^2 + 2x - 2y - 7 = 0$

Comparing both the circles with

$x^2 + y^2 + 2gx + 2fy + c = 0$

$2g = 2 \quad , \quad 2f = -2 \qquad c = -7$

$g = 1 \quad , \quad f = -1 \qquad c = -7$

$Centre = c_1(-g\ ,f) = c_1(-1\ ,1)$

$Radius = r_1 = \sqrt{g^2 + f^2 - c}$

$$= \sqrt{1^2 + (-1)^2 + 7 = 3}$$

$S_2: x^2 + y^2 - 6x + 4y + 9 = 0$

$2g = -6 \quad , \quad 2f = 4 \quad , \quad c = 9$

$g = -3 \quad , \quad f = 2 \quad , \quad c = 9$

$Centre = c_2(-g\ ,-f) = c_2(3\ ,-2)$

$Radius = r_2 = \sqrt{g^2 + f^2 - c}$

$$= \sqrt{(-3)^2 + 2^2 - 9} = 2$$

$Two\ circles\ touch\ each\ other\ etermally\ if$

$|c_1c_2| = r_1 + r_2$

$\sqrt{(-1-3)^2 + (1+2)^2} = 3 + 2$

$\sqrt{16+9} \ = 5$

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$5 = 5$ Proves

$So\ circles\ S_1 and\ S_2\ touch\ each\ other\ extermally$

Q.5

$S_1: \quad x^2 + y^2 + 2x - 8 = 0$

Comparing both the circles with

$x^2 + y^2 = 2gx + 2fy + c = 0$

$2g = 2 \quad , \quad 2f = 0 \quad c = -8$

$g = 1 \quad , \quad f = 0 \quad c = -8$

$Centre = c_1(-g\ , -f) = c_1(-1\ , 0)$

$Radius = r_1 = \sqrt{g^2 + f^2 - c}$

$r_1 \quad = \sqrt{1 + 0 + 8} \quad = 3$

$S_2: x^2 + y^2 - 6x + 6y - 46 = 0$

$2g = -6 \quad , \quad 2f = -6 \quad , \quad c = -46$

$g = -3 \quad , \quad f = 3 \quad , \quad c = -46$

$Centre = c_2(-g\ , -f) = \ c_2(3\ , -3)$

$Radius = r_2 = \sqrt{g^2 + f^2 - c}$

$r_2 = \sqrt{9 + 9 + 46} = 8$

$Two\ circles\ touch\ each\ other\ internally\ of$

$|c_1c_2| = |r_1r_2|$

$\sqrt{(-1-3)^2 + (0+3)^2} = 8$

$\sqrt{16 + 9} = 5 \Longrightarrow \quad 5 = 5 \qquad (proved)$

$So\ circles\ S_1 and\ S_2$touch each other internally

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Q.6

$A(1\,,-3)$

$l: x - y - 4 = 0$

$Radious = 2$

$Centre: C(-g\,,-f)$

$Let\ required\ equation\ of\ circle$

$x^2 + y^2 + 2gx + 2fy + c = 0$

$Since\ point\ A\ lies\ on\ circle\ (1), therefore$

$At\ A(1\,,-3)$

$(1)^2 + (-3)^2 + 2g(1) + 2f(-3) + c = 0$

$1 + 9 + 2g - 6f + c = 0$

$2g - 6f + c + 10 = 0 \quad \ldots.(1)$

$As \quad radius = 2, so$

$\sqrt{g^2 + f^2 - c} = 2$

$\Longrightarrow g^2 + f^2 - c - 4 = 0$

$Eq(1) + Eq(2)$

$2g - 6f + c + 10 = 0$

$\frac{x^2+f^2-c-4=0}{2g-6f+f^2+6=0} \quad \ldots.(3)$

$Slope\ of\ l = m_1 = -\frac{1}{-1} = 1$

$Slope\ of\ \overline{AC} = m_2 = \frac{-3+f}{1+g}$

$As\ \overline{AC} \perp l \begin{cases} \therefore Radial\ segment\ at\ A\ and\ tangent \\ \qquad at\ A\ are\ perpendicular \end{cases}$

$\Longrightarrow m_1 \times m_{2=-1}$

$1\left(\frac{-3+1}{1+g}\right) = -1$

$-3 + f = -1 - g$

$\Longrightarrow \quad f + g - 2 = 0$

$\Longrightarrow \quad g = 2 - f \quad \ldots..(4) put\ in\ (3)$

$2(2-f) - 6f + (2-f)^2 + f^2 + 6 = 0$

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$4 - 2f - 6f + 4 + f^2 - 4f + f^2 + 6 = 0$

$2f^2 - 12f + 14 = 0$

$f^2 - 6f + 7 = 0$

By using quadratic formula

$a = 1\,, b = \ -6\,, c = 7$

$f = \frac{6+\sqrt{(-6)^1 - 4(1)(7)}}{2(1)}$

$f = \frac{6+\sqrt{36-28}}{2} = \frac{6 \pm \sqrt{8}}{2}$

$f = \frac{6+2\sqrt{2}}{2} = 3 \pm \sqrt{2}$

$When\ f\ = 3 + \sqrt{2}$

$Eq(4) \Longrightarrow g = 2 - 3 - \sqrt{2} \Longrightarrow g = -1 - \sqrt{2}$

$Eq(1) \Longrightarrow 2\left(-1 - \sqrt{2}\right) - 6\left(3 + \sqrt{2}\right) + c + 10 = 0$

$-2 - 2\sqrt{2} - 18 - 6\sqrt{2} + c + 10 = 0$

$-20 - 8\sqrt{2} + 10 + c = 0$

$\Longrightarrow\ -8\sqrt{2} - 10 + c = 0$

$c = 10 + 8\sqrt{2}$

Putting values I eq (1)

$x^2 + y^2 + 2\left(-1 - \sqrt{2}\right)x + 2\left(3 + \sqrt{2}\right)y + 10 + 8\sqrt{2} = 0$

$When\ f\ = 3 - \sqrt{2}$

$Eq(4) \Longrightarrow g = 2 - 3 + \sqrt{2} \Longrightarrow g = -1 + \sqrt{2}$

$Eq(1) \Longrightarrow 2\left(-1 + \sqrt{2}\right) - 6\left(3 - \sqrt{2}\right) + c + 10 + 0$

$-2 + 2\sqrt{2} - 18 + 6\sqrt{2} + c + 10 = 0$

$8\sqrt{2} - 10 + c = 0$

$\Longrightarrow c = 10 - 8\sqrt{2}$

$Putting\ values\ in\ eq(1)$

$x^2 + y^2 + 2\left(-1 + \sqrt{2}\right)x + 2\left(3 - \sqrt{2}\right)y + 10 - 8\sqrt{2} = 0$

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Q.7

(a) The equation of the circle is given as:

$$x^2 + y^2 - 6x + 10y + 9 = 0$$

To write this equation in the standard form

$$(x-h)^2 + (y-k)^2 = r^2 \quad \rightarrow (i)$$

Now,

$$x^2 - 6x + y^2 + 10y = -9$$

$$(x^2 - 6x + 9) + (y^2 + 10y + 25) = -9 + 9 + 25$$

$$(x-3)^2 + (y+5)^2 = 25 \quad \rightarrow (ii)$$

Now,

Comparing the equation (i) and (ii)

$\Longrightarrow \quad Center\ (h,k) = (3,5)$

$\Longrightarrow \quad radius\ (r) = 5$

So, the center of the circle is (3,-5) and the radius is 5.

(b) The line y = Kx intersect the circle at two distinct points.

Substituting this into the circle equation, we get

$$x^2 + (kx)^2 - 6x + 10(kx) + 9 = 0$$

$$x^2(1+k^2) - 6x + 10kx + 9 = 0$$

Here,

$$a = 1 + k^2, b = 4(2.5)k, c = 9$$

Now,

$$b^2 - 4ac \geq 0$$

The discriminant must be greater than or equal to zero

$$\left(4(2.5k)\right)^2 - 4(1+k^2)(a) \geq 0$$

$$100k^2 - 36 - 36k^2 \geq 0$$

$$64k^2 \geq 36$$

$$k^2 \geq \frac{9}{16}$$

$$k \geq \frac{3}{4} \ or\ k \leq \frac{-3}{4}$$

So, the range of values for k is

$$k \geq \frac{3}{4} \ or\ k \leq \frac{-3}{4}$$

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Q.8

(a) Given equation of circle

$$x^2 - 8x + y^2 + 6y = -16$$

$$(x^2 - 8x + 16) + (y^2 + 6y + 9) = -16 + 16 + 9$$

$$(x - 4)^2 + (y + 3)^2 = 9$$

$$(x - 4)^2 + (y + 3)^2 = (3)^2 \rightarrow (i)$$

Using standard form of circle

$$(x - h)^2 + (y - k)^2 = (r)^2 \rightarrow (ii)$$

Comparing (i) and (ii) , we get

$$Center\ (h, k) = (4, -3)$$

(b) $radius\ (r) = 3$

(c) Consider the line y = kx , where k is a constant

Substituting y = kx into the circle equation

$$x^2 + (kx)^2 - 2x + 4(kx) + 5 = 0$$

$$x^2(1 + k^2) + 2x(2k) + 5 = 0$$

For real solution , the discriminant must be greater than or equal to zero

$$\left(2(2k)\right)^2 - 4(1 + k^2)(5) \geq 0$$

Simplify and solve for k:

$$16k^2 - 20(1 + k^2) \geq 0$$

$$16k^2 - 20 - 20k^2 \geq 0$$

$$-4k^2 - 20 \geq 0$$

$$k^2 + 5 \leq 0$$

This inequality has no real solution for k, meaning that any line y = kx where

K is a constant will not intersect the circle C_4 , except the ease when the line Coincides with the circle.

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