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TUITION

Phone: +442081445350

www.chemistryonlinetuition.com

Email:asherrana@chemistryonlinetuition.com

PURE MATH

ALGEBRA AND FUNCTION

Level & Board	EDEXCEL (A-LEVEL)
TOPIC:	CIRCLES
PAPER TYPE:	SOLUTION - 6
TOTAL QUESTIONS	8
TOTAL MARKS	64

Q1.

$S_1: x^2 + y^2 + 2x - 2y - 7 = 0$

Comparing both the circles with

$x^2 + y^2 + 2gx + 2fy + c = 0$

$2g = 2 \quad , \quad 2f = -2 \qquad c = -7$

$g = 1 \quad , \quad f = -1 \qquad c = -7$

$Centre = c_1(-g\,, f) = c_1(-1\,, 1)$

$Radius = \; r_1 = \sqrt{g^2 + f^2 - c}$

$= \sqrt{1^2 + (-1)^2 + 7 = 3}$

$S_2: x^2 + y^2 - 6x + 4y + 9 = 0$

$2g = -6 \quad , \quad 2f = 4 \quad , \; c = 9$

$g = -3 \quad , \quad f = 2 \quad , \; c = 9$

$Centre = c_2(-g\,, -f) = c_2(3\,, -2)$

$Radius = r_2 = \sqrt{g^2 + f^2 - c}$

$= \sqrt{(-3)^2 + 2^2 - 9} = 2$

$Two\ circles\ touch\ each\ other\ etermally\ if$

$|c_1c_2| = r_1 + r_2$

$\sqrt{(-1-3)^2 + (1+2)^2} = 3 + 2$

$\sqrt{16 + 9} \;\; = 5$

$5 = 5$ Proves

$So\ circles\ S_1 and\ S_2\ touch\ each\ other\ extermally$

Q2.

(i) The general form of a circle equation

$$x^2 + y^2 + 2ax + 2by + c = 0 \ \rightarrow (ii)$$

Given equation

$$x^2 + y^2 + 18x - 2y + 30 = 0 \ \rightarrow (ii)$$

Comparing equation (i) and (ii)

$$2a = 18 \ \rightarrow \quad a = 9$$

$$2b = -2 \ \rightarrow \quad b = -1$$

$$c = 30$$

So, the Centre of circle C is (-9,1)

The given point P(-5,7)

$\Longrightarrow$

$$r = \sqrt{(x_P - a)^2 + (y_P - b)^2}$$

$$r = \sqrt{(-5 + 9)^2 + (7 - 1)^2}$$

$$r = \sqrt{16 + 36}$$

$$r = \sqrt{52}$$

$$r = \sqrt{4 \times 13}$$

$$r = 2\sqrt{13}$$

Now, The equation of the tangent at the point (-5,7) is given by:

$$x(x_P) + y(y_P) = r^2$$

$$(x + 5) + (y - 7) = \left(2\sqrt{13}\right)^2$$

$$x + y - 2 = 4(13)$$

$$x + y = 52 - 2$$

$$x + y - 50 = 0$$

So, the required equation of the tangent line is x + y – 50 = 0

(ii) where,

$$a = \frac{-8}{2} = -4 \; and \; b = \frac{12}{2} = 6$$

$\Longrightarrow$ the Centre is $(-9\,,6)$

So,

$$\sqrt{(-4)^2 + (6)^2 - k} < 4$$

$\Longrightarrow$

$$16 + 36 - k < 4$$

$$52 - k < 16$$

$$-k < -36$$

$$k > 36$$

Therefore,

The range of possible values for k is $k > 36$

Q3.

The distance between two points x_1, y_1 and x_2, y_2 in a plane is given by the

Distance formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = |PQ| = \sqrt{(8-3)^2 + (4-2)^2}$$

$$PQ = \sqrt{25 + 4}$$

$$PQ = \sqrt{29}$$

Q.4

Part (a):

(i) Using the general form of the equation of a circle is:

$(x-h)^2+(y-k)^2=r^2 \ \rightarrow (i)$

Using given equation of the circle

$x^2+y^2-4x+8y-8=0$

$\Longrightarrow$

$(x^2-4x)+(y^2+8y)-8=0$

$\Longrightarrow$

$(x^2-4x+4)+(y^2+8y+16)-8-4-16=0$

$\Longrightarrow$

$(x-2)^2+(y+4)^2-28=0 \ \rightarrow (ii)$

So, the center of the circle (h,k) is (2,-4)

(ii) The radius r cab be found from the equation $(x-h)^2+(y-k)^2=r^2$

$\Longrightarrow \quad r^2=28$

$\Longrightarrow \quad \sqrt{r^2}=\sqrt{28}$

$\Longrightarrow \quad r=\sqrt{4\times 7}$

$\Longrightarrow \quad r=2\sqrt{7}$

Part (b):

The line r = k is a vertical line for this line to be tangent to the circle, the distance between the center of the center and the line should be equal to the radius of the circle. The distance between a point (a , b) and the line z = k is given by $|a-k|$ So, for the tangent condition.

$|2-k| = 2\sqrt{7}$

$\Longrightarrow \quad 2-k = \pm 2\sqrt{7}$

$\Longrightarrow \quad 2-k = 2\sqrt{7} \quad or \;\; 2-k = -2\sqrt{7}$

$\Longrightarrow \quad k = 2-2\sqrt{7} \quad or \;\; k = 2+2\sqrt{7}$

So, the range of values for k is $2-2\sqrt{7} \le k \le 2+2\sqrt{7}$

Q.5

The equation of circle by standard form is

$\Longrightarrow \quad (x-h)^2 + (y-k)^2 = r^2$

$\Longrightarrow \quad (x-5)^2 + \left(y-(-2)\right)^2 = (4)^2$

$\Longrightarrow \quad (x-5)^2 + (y+2)^2 = 16$

$\Longrightarrow \quad x^2 + 25 - 10x + y^2 + 4 - 4y - 16 = 0$

$\Longrightarrow \quad x^2 + y^2 - 10x - 4y + 29 - 16 = 0$

$\Longrightarrow \quad x^2 + y^2 - 10x - 4y + 13 = 0$

Q6.

Part (a):

Given equation of circle

$x^2 - 8x + y^2 + 6y = -16$

$$(x^2 - 8x + 16) + (y^2 + 6y + 9) = -16 + 16 + 9$$

$$(x - 4)^2 + (y + 3)^2 = 9$$

$$(x - 4)^2 + (y + 3)^2 = (3)^2 \rightarrow (i)$$

Using standard form of circle

$$(x - h)^2 + (y - k)^2 = (r)^2 \rightarrow (ii)$$

Comparing (i) and (ii) , we get

$$Center\ (h, k) = (4, -3)$$

Part (b):

$$radius\ (r) = 3$$

Part (c):

Consider the line y = kx , where k is a constant

Substituting y = kx into the circle equation

$$x^2 + (kx)^2 - 2x + 4(kx) + 5 = 0$$

$$x^2(1 + k^2) + 2x(2k) + 5 = 0$$

For real solution , the discriminant must be greater than or equal to zero

$$\left(2(2k)\right)^2 - 4(1 + k^2)(5) \geq 0$$

Simplify and solve for k:

$$16k^2 - 20(1 + k^2) \geq 0$$

$$16k^2 - 20 - 20k^2 \geq 0$$

$$-4k^2 - 20 \geq 0$$

$$k^2 + 5 \leq 0$$

This inequality has no real solution for k, meaning that any line y = kx where

K is a constant will not intersect the circle C_4 , except the ease when the line

Coincides with the circle.

Q7.

Part (a):

The equation of the circle is $x^2+y^2=r^2$, and since it touches the x-axis, its center is at the point (0, r) in the 1st quadrant. Substitute $x= y+5$ into the equation of the circle:

$$(y+5)^2+y^2=r^2$$

Expand and simplify:

$$2y^2+10y+25=2x^2+10x+25=r^2$$

Now, move all terms to one side of the equation:

$$2y^2-10y+25-r^2=2x^2-10x+25-r^2=0$$

This matches the desired equation

$$2y^2-10y+25-r^2=2x^2-10x+25-r^2=0.$$

Part (b):

For line l to be a tangent to circle C, the discriminant of the quadratic equation representing the points of intersection must be zero.

The discriminant is given by b^2-4ac, where the quadratic equation is

$$ax^2+bx+c=0.$$

In this case, the discriminant is $(-10)^2-4(2)(25-r^2)$, and it must be equal to zero.

$$100-4(50-2r^2)=0$$

Simplify and solve for r:

$$8r^2=100$$

$$r^2=12.5$$

$$r=\sqrt{12.5}$$

Therefore, the possible value for r is approximately 3.54.

Q8.

Part (a):

The equation of the circle is $x^2 + y^2 = r^2$, and since it touches the x-axis, its center is at the point (0, -r) in the 3rd quadrant. Substitute $2x + 5y = 15$ into the equation of the circle:

$$x^2 + (15 - 2x)^2 = r^2$$

Expand and simplify:

$$x^2 + 225 - 60x + 4x^2 = r^2$$

Combine like terms:

$$5x^2 - 60x + 225 = r^2$$

Now, move all terms to one side of the equation:

$$5x^2 - 60x + 225 - r^2 = 0$$

This matches the desired equation $5x^2 - 60x + 225 - r^2 = 0$.

Part (b):

For line l to be a tangent to circle C, the discriminant of the quadratic equation representing the points of intersection must be zero. The discriminant is given by $b^2 - 4ac$, where the quadratic equation is $ax^2 + bx + c = 0$.

In this case, the discriminant is $(-60)^2 - 4(5)(225 - r^2)$, and it must be equal to zero.

$$3600 - 4(5)(225 - r^2) = 0$$

Simplify and solve for r:

$$3600 - 4500 + 16r^2 = 0$$

$$16r^2 = 900$$

$$r^2 = 225$$

Therefore, the possible value for r is r = 15.

