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PURE MATH

ALGEBRA AND FUNCTION

Level & Board	EDEXCEL (A-LEVEL)
TOPIC:	FUNCTIONS
PAPER TYPE:	SOLUTION - 1
TOTAL QUESTIONS	8
TOTAL MARKS	38

1.

Range of f:

To find the range of f(x), we can complete the square on the quadratic expression:

$$f(x) = x^2 + 2x + 1$$
$$= (x + 1)^2$$

The square of any real number is non – negative, so (x +1)2 is always greater than or equal to zero. Therefore, the range of f(x) is all non – negative real numbers:

Range of $f = \{y \in \mathbb{R} \mid y \geq 0\}$

Inverse Function $f^{-1}(x)$ and its Domain:

To find the inverse function $f^{-1}(x)$, interchange x and y in the equation $f(x) = (x +1)^2$ and solve for y:

Taking the square root of both sides:

$$\sqrt{x} = y = 1$$

Solving for y:

$$y = \sqrt{x} - 1$$

So, the inverse function $f^{-1}(x)$ is given by $f^{-1}(x) = \sqrt{x} - 1$.

Now, let's consider the domain of $f^{-1}(x)$. The square root is defined only for non – negative real numbers, so $\sqrt{x}$ is defined when $x \geq 0$. Also, subtracting 1 does not affect this domain restriction. Therefore, the domain of $f^{-1}(x)$ is $x \geq 0$.

In summary:

$f^{-1}(x) = \sqrt{x} - 1$

Domain of $f^{-1}(x)$: $x \geq 0$

2.

Range of g:

The range of g(x) can be determined by analyzing the linear expression 3x – 4. Since the coefficient of x is positive, the function is increasing.

Also,

There are no restrictions on x other than $x \geq 2$. Therefore, the range of g(x) is all numbers for $x \geq 2$:

Inverse Function $g^{-1}(x)$ and its Domain:

To find the inverse function $g^{-1}(x)$, interchange x and y in the equation g(x) = 3x – 4 and solve for y:

$$x = 3y - 4$$

Solve for y:

$$y = \frac{x+4}{3}$$

So, the inverse function $g^{-1}(x)$ is given by $g^{-1}(x)$ = = $\frac{x+4}{3}$ is defined for all real numbers. Therefore, the domain of $g^{-1}(x)$ is $\mathbb{R}$.

In summary:

$$g^{-1}(x) = \frac{x+4}{3}$$

Domain of $g^{-1}(x)$: $\mathbb{R}$

3)

Range of h:

The function $h(x) = e^x$ is the exponential function, and its range is all positive real numbers

Range of h = $\{y \in \mathbb{R} | y > 0\}$

Inverse Function h-1(x) and its Domain:

To find the inverse function $h^{-1}(x)$, interchange x and y in the equation $h(x) = e^x$ and solve for y:

$$x = e^y$$

Taking the natural logarithm of both sides:

$$In(x) = y$$

So, the inverse function $h^{-1}(x)$ is given by $h^{-1}(x)$ = In(x).

Now, let's consider the domain of $h^{-1}(x)$. The natural logarithm is defined only for positive real numbers, so the domain of $h^{-1}(x)$ is $x > 0$.

In summary

$$h^{-1}(x) = In(x)$$

Domain of $^{h-1}(x)$: $x > 0$

4)

Range of p:

To find the range of p(x), we can complete the square on the quadratic expression:

$p(x) = x^2 – 4x + 3$

$= (x – 2)^2 – 1$

The square of any real number is non – negative, so (x -2)2 is always greater than or equal to zero. Therefore, the range of p(x) is all real numbers except when (x – 2)2 = 0, which occurs when x = 2. So, the range of p is all real numbers except – 1:

Range of p = $\{y \epsilon \mathbb{R} \mid y \neq -1\}$

Inverse Function $p^{-1}(x)$ and its Domain:

To find the inverse function $p^{-1}(x)$, interchange x and y in the equation

$p(x) = (x -2)^2 - 1$

and solve for y:

$x = (y – 2)^2 – 1$

Adding 1 to both sides:

$x + 1 = (y – 2)^2$

Taking the square root of both sides:

$\sqrt{x+1}$ y- 2

Solve for y:

$Y = \sqrt{x+1} + 2$

So, the inverse function p-1(x) is given by p-1(x) = $\sqrt{x+1}$ + 2.

Now, let's consider the domain of p-1(x). The square root is defined only for non – negative real numbers, so $\sqrt{x+1}$ is defined when $x + 1 \geq 0$.

Therefore, the domain of p-1(x) is $x \geq -1$.

In summary:

p-1(x) = $\sqrt{x+1}$ + 2

Domain of p-1(x): $x \geq - 1$

5.

Range of r:

The logarithmic function log2(x + 4) is defined for x > -4. The range of r(x) will be all real numbers because the logarithm is defined for positive real numbers.

Range of r = $\{y \in \mathbb{R}\}$

Inverse Function $r^{-1}(x)$ and its Domain:

To find the inverse function $x^{-1}(x)$, interchange x and y in the equation

$r(x) = \log_2(x + 4)$

and solve for y:

$x = \log_2(y + 4)$

rewrite in exponential form:

$2^x = y + 4$

Solve for y:

$Y = 2^x - 4$

So, the inverse function $r^{-1}(x)$ is given by $r^{-1}(x) = 2^x - 4$.

Now, let's consider the domain of r-1(x). The exponential function 2^x is defined for all real numbers, and subtracting 4 does not impose any additional restrictions.

Therefore, the domain of $r^{-1}(x)$ is $\mathbb{R}$.

6.

Range of t:

The cubic function $t(x) = x^3 + 2x^2 - x - 3$ is defined for all real numbers. To find the range, we can analyze its behavior. The leading term is x^3, which means the function increases without bound as x approaches positive or negative infinity. Therefore, the range of t(x) is all real numbers:

Range of t = $\{y \in \mathbb{R}\}$

Inverse Function $t^{-1}(x)$ and its Domain:

Finding the inverse function for a cubic function can be challenging due to its non – one – to – one nature. Cubic functions are not always invertible, and in this case, we may not be able to find a simple algebraic expression for $t^{-1}(x)$.

in cases where the inverse function is not easily expressed algebraically, it may be represented graphically or numerically.

If we attempt to find the inverse function, we swap x and y and try to solve for y:

$$x = y^3 + 2y^2 - y - 3$$

However, solving this cubic equation for y is generally complex may involve numerical method.

However, solving this cubic equation for y is generally complex and may involve numerical method.

So, for this example, we may not provide a simple algebraic expression for the inverse function. The focus would be on the cubic function itself and its behavior.

In summary:

Range of t = $\{y \in \mathbb{R}\}$

The inverse function $t^{-1}(x)$ may not have a simple algebraic expression.

7.

Range of f:

The function $v(x) = \log_3(|x| + 2)$ involves a logarithm with a base of 3.

The argument of the logarithm is $|x| + 2$, and $|x| + 2$ is always greater than or equal to 2.

The logarithm is defined for positive arguments, so the minimum value of $f(x)$ is $\log_3(2)$,

which is greater than O. therefore, the range of $f(x)$ is all real numbers, excluding negative values.

so, the range of f is $\{y \in \mathbb{R} | \ y > 0\}$

8.

Range of g:

The function g(x) = $\frac{1}{x-1}$ is defined for all real numbers except x – 1. As x approaches 1 from the left (x → 1^-), g(x) goes to negative infinity, and as x approaches 1 from the right (x → 1^-), g(x) goes to positive infinity. Therefore, the range of g(x) is all real numbers, excluding O.

So, the range of g is $\{y \epsilon \mathbb{R} | \ y \neq 0 \ \}$

Inverse Function g-1(x) and its Domain:

To find the inverse function g-1(x), interchange x and y in the equation g(x) - $\frac{1}{x-1}$ for y:

x = $\frac{1}{x-1}$

solving for y:

$$y = \frac{1}{x} + 1$$

so the inverse function $g^{-1}(x)$ is given by

$g^{-1}(x) = \frac{1}{x} + 1$

Now, let's consider the domain of g-1(x). The inverse function is defined for all real numbers except x - 0 (since division by zero is undefined).

In summary:

$g^{-1}(x) = \frac{1}{x} + 1$

Domain of $g^{-1}(x)$: $\mathbb{R}$, exchanging x - 0

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