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PURE MATH

ALGEBRA AND FUNCTION

Level & Board	EDEXCEL (A-LEVEL)
TOPIC:	FUNCTIONS
PAPER TYPE:	SOLUTION - 2
TOTAL QUESTIONS	8
TOTAL MARKS	38

1)

Range of h:

The function h(x) = 2^{x-1} is an exponential function with a base of 2. As x varies over all real numbers, the exponential function h(x) takes on all positive values. So, the range of h is $\{y \in \mathbb{R} \mid y > 0\}$.

Inverse Function h-1(x) and its Domain:

To find the inverse function h-1(x), interchange x and y in the equation h(x) – 2^{x-1} and solve for y:

x = 2^{y-1}

Taking the logarithm base 2 of both sides:

$\log_2(x) = y - 1$

Solving for y:

$y = \log_2(x) + 1$

So, the inverse function $h^{-1}(x)$ is given by:

h-1(x) = $\log_2(x) + 1$

Now, let's consider the domain of $h^{-1}(x)$. The logarithmic function is defined only for positive real numbers, so the domain of $h^{-1}(x)$ is $\{x \in \mathbb{R} \mid x > 0\}$.

In summary:

$h^{-1}(x) = \log_2(x) + 1$

Domain of $h^{-1}(x)$: $\{x \in \mathbb{R} \mid x > 0\}$.

2)

Range of p:

The quadratic function p(x) = (x – 3)2 + 4 has its veretex at (3,1),, and the square term (x – 3)2 is always non – negative. Therefore, the minimum value of p(x) is 4, and as x varies over all real numbers, p(x) takes on all values greater than or equal to 4. So, the range of p is

$\{x \in \mathbb{R} \mid x \geq 4\}$.

Inverse Function $p^{-1}(x)$ and its Domain:

To find the inverse function $p^{-1}(x)$, interchange x and y in the equation p(x) = $(x - 3)^2 + 4$

and solve for y:

$$y = \pm\sqrt{x-4} + 3$$

Since p(x) is an upward – opening parabola, only the positive square root is considered to keep it as a function. So, the inverse function $p^{-1}(x)$ is given by:

$p^{-1}(x) = \sqrt{x-4} + 3$

Now, let's consider the domain or p-1(x). The square root function is defined only for non – negative real numbers, so the domain of $p^{-1}(x)$ is $\{x \in \mathbb{R} \mid x \geq 4\}$.

In summary:

$p^{-1}(x) = \sqrt{x-4} + 3$

Domain of $p^{-1}(x)$: $\{x \in \mathbb{R} \mid x \geq 4\}$

3)

Range of m:

The cubic function m(x) = x3 – 4x is defined for all real numbers. To find the range, we can analyze its behavior. The leading term is x3, which means the function increases without bound as x approaches positive or negative infinity. Therefore, the range of m(x) is all real numbers: Range of m = $\{y \in \mathbb{R}\}$

Inverse Function $m^{-1}(x)$ and its Domain:

Finding the inverse function for a cubic function can be challenging due to its non – one – to – one nature. Cubic functions are not always invertible, and in this case, we may not able to find a simple algebraic expression for $m^{-1}(x)$.

In case where the inverse functions is not easily expressed algebraically, it may be represented graphically or numerically.

If we attempt to find the inverse function, we swap x and y and try to solve for y:

$x = y^3 - 4y$

However, solving this cubic equation for y is generally complex and may involve numerical methods.

So, for this example, we may not provide a simple algebraic expression for the inverse function. The focus would be on the cubic function itself and its behavior.

In summary:

Range of m = $\{y \epsilon \mathbb{R}\}$

The inverse function m-1(x) may not have a simple algebraic expression.

4)

Range of n:

The function n(x) = $\frac{2x}{x-4}$ is a rational function. As x approaches 4, the denominator

(x -4) approaches O, leading to an undefined value. Therefore, the function has a vertical asymptote at x = 4. Apart from that, the function can take any real value.

So, the range of n(x) is $\{x \epsilon \mathbb{R} \,|x \neq 0\}$ since the function is undefined when x = 4.

Inverse Function $n^{-1}(x)$ and its Domain:

To find the inverse function n-1(x). interchange x and y in the equation n(x) = $\frac{2x}{x-4}$ and solve for y:

(x) = $\frac{2x}{x-4}$

Now, solving for y:

y = $\frac{4x}{x-2}$

So, the inverse function $n^{-1}(x)$ is given by:

n-1(x) = $\frac{4x}{x-2}$

So, the inverse function $n^{-1}(x)$ is given by:

$n^{-1}(x) = \frac{4x}{x-2}$

Now, let's consider the domain of $n^{-1}(x)$. The inverse function is defined for all real numbers except where the original function is undefined. Since n(x) undefined at x = 4, the domain of $n^{-1}(x)$ is $\mathbb{R}$, excluding x = -2 (where the denominator (x + 2) is equal to 0).

In summary:

n-1(x) = $\frac{4x}{x-2}$

Domain of n-1(x): $\mathbb{R}$, excluding x = - 2.

5)

Ranger of p:

The function p(x) = $\sqrt{x+3}$ is a square root function with a shift of 3 unit to the left. The square root a defined only for non – negative values, and the shift ensures that the function is always, defined for x $\geq$ - 3. As x varies over all real numbers greater than or equal to – 3, the expression $\sqrt{x+3}$ covers all non – negative real numbers. So, the range of p(x) is $\{y \epsilon \mathbb{R} \,|y \geq 0\}$.

Inverse Function p-1(x) and its Domain:

To find the inverse function $p^{-1}(x)$, interchange x and y in the equation p(x) = $\sqrt{x+3}$ and solve for y:

x= $\sqrt{y+3}$

Squaring both sides:

$x^2 = y + 3$

Solving for y:

$y = x^2 – 3$

So, the inverse function $p^{-1}(x)$ is given by:

$p^{-1}(x) = x^2 – 3$

Now, let's consider the domain of p-1(x). The inverse function is defined for all real numbers, so the domain of $p^{-1}(x)$ is $\mathbb{R}$.

In summary:

$p^{-1}(x) = x^2 – 3$

Domain of $p^{-1}(x)$: $\mathbb{R}$

6)

Range of y:

The function $q(x) = 2^x$ is an exponential function with a base of 2. As x varies over all real numbers, the expression 2^x covers all positive real numbers.

So, the range of q(x) is $\{y \epsilon \mathbb{R} \,|y > 0\}$.

Inverse Function $q^{-1}(x)$ and its Domain:

To find the Inverse Function $q^{-1}(x)$, interchange x and y in the equation q(x) = 2x and solve for y:

$x = 2^y$

Taking the logarithm base 2 of both sides:

$\log_2(x) = y$

So, the inverse function $y^{-1}(x)$ is given by:

$q^{-1}(x) = \log_2(x)$

Now, let's consider the domain of y-1(x). The logarithmic function only for positive real numbers, so the domain of $q^{-1}(x)$ is $\{x \epsilon \mathbb{R} \,|x > 0\}$.

In summary:

$q^{-1}(x) = \log_2(x)$

Domain of $q^{-1}(x)$: $\{x \epsilon \mathbb{R} \,|x > 0\}$.

7)

Range of r:

The function $r(x) = (x – 1)^3$ is a cubic function. As x varies over all real numbers, the expression (x – 1)3 can take on all real values.

So, the range of r(x) is $\{y \epsilon \mathbb{R}\}$.

Inverse Function r-1(x) and its Domain:

To find the inverse function $r^{-1}(x)$, interchange x and y In the equation $r(x) = (x - 1)^3$ and solve for y:

$x = (y - 1)^3$

Taking the cube root of both sides:

$y - 1 = \sqrt[3]{x}$

Solving for y:

$y = \sqrt[3]{x} + 1$

So, the inverse function $r^{-1}(x)$ is given by:

$r^{-1}(x) = \sqrt[3]{x} + 1$

Now, let's consider the domain of $r^{-1}(x)$. The cube root function is defied for all real numbers. So, the domain of $r^{-1}(x)$ is $\mathbb{R}$.

In summary:

$r^{-1}(x) = \sqrt[3]{x} + 1$

Domain of r-1(x): $\mathbb{R}$

8)

Range of s:

The function $s(x) = -\lceil r + 2 \rceil$ is an absolute value function with a reflection, and the

negative sign ensures that the output is always negative. As x varies over all real numbers, the expression $= -\lceil r + 2 \rceil$ covers all real numbers but is always negative.

So, the range of s(x) is $\{x \in \mathbb{R} \mid x < 0\}$.

Inverse Function $s^{-1}(x)$ and its Domain:

To find the inverse function $s^{-1}(x)$, interchange x and y in the equation

$s(x) = -\lceil r + 2 \rceil$ and solve for y:

$x = -\lceil r + 2 \rceil$

Dividing both sides by – 1 to isolate the absolute value:

$- x = \lceil r + 2 \rceil$

Now, consider two cases:

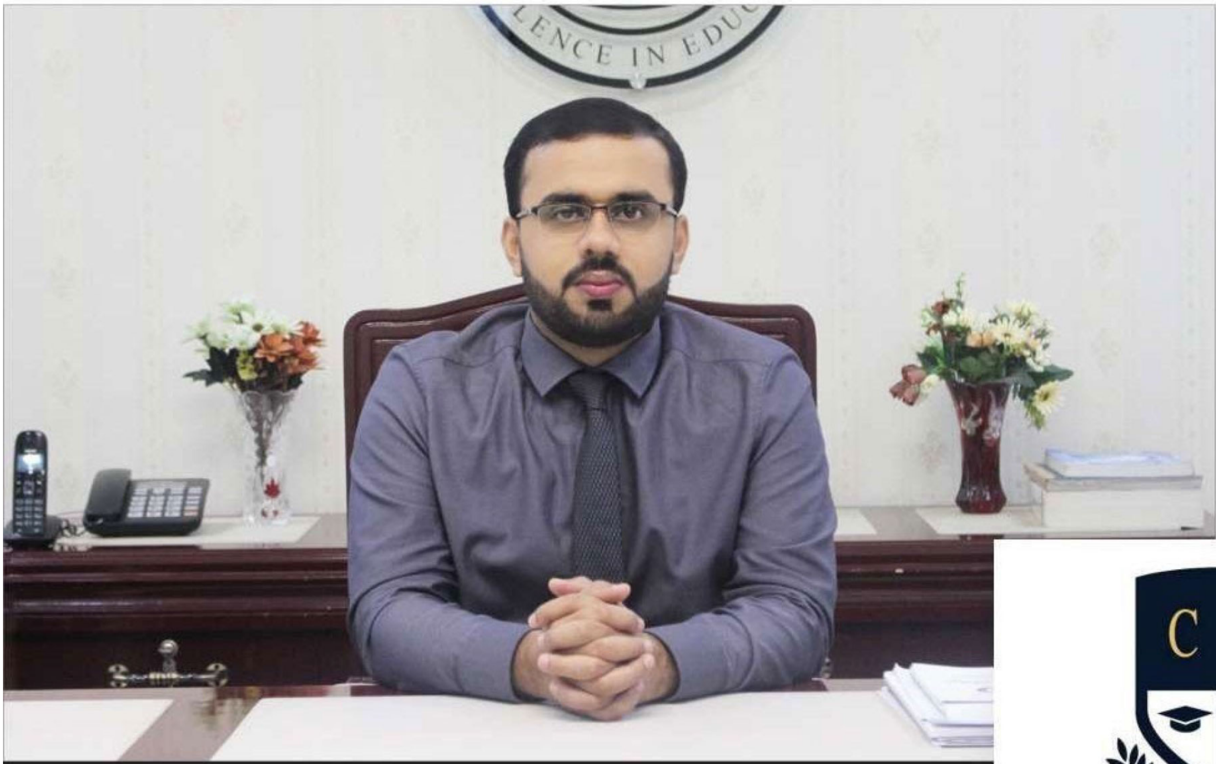
1. $x \geq 0$: - x = y + 2, so y = - x – 2.
2. x < 0: - x = - (y + 2), so y = - x – 2.
   So, the inverse function $s^{-1}(x)$ is given by:
   $s^{-1}(x) = - x – 2$

Now, let's consider the domain of $s^{-1}(x)$. The inverse function is defined for all real numbers, so the domain of $s^{-1}(x)$ is $\mathbb{R}$.

in summary:

$s^{-1}(x) = - x – 2$

Domain of $x^{-1}(x)$: $\mathbb{R}$



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