Phone: +442081445350

www.chemistryonlinetuition.com

Email:asherrana@chemistryonlinetuition.com

PURE MATH

ALGEBRA AND FUNCTION

Level & Board	EDEXCEL (A-LEVEL)
TOPIC:	FUNCTIONS
PAPER TYPE:	SOLUTION - 3
TOTAL QUESTIONS	8
TOTAL MARKS	38

1.

Range of f:

To find the range of $f(x)$, we can complete the square on the quadratic expression:

$$f(x) = x^2 + 2x + 1$$
$$= (x + 1)^2$$

The square of any real number is non – negative, so (x +1)2 is always greater than or equal to zero. Therefore, the range of $f(x)$ is all non – negative real numbers:

Range of $f = \{y \epsilon \mathbb{R} \mid y \geq 0\}$

Inverse Function $f^{-1}(x)$ and its Domain:

To find the inverse function $f^{-1}(x)$, interchange x and y in the equation $f(x) = (x + 1)^2$ and solve for y:

Taking the square root of both sides:

$$\sqrt{x} = y = 1$$

Solving for y:

$$y = \sqrt{x} - 1$$

So, the inverse function $f^{-1}(x)$ is given by $f^{-1}(x) = \sqrt{x} - 1$.

Now, let's consider the domain of $f^{-1}(x)$. The square root is defined only for non – negative real numbers, so $\sqrt{x}$ is defined when $x \geq 0$. Also, subtracting 1 does not affect this domain restriction. Therefore, the domain of $f^{-1}(x)$ is $x \geq 0$.

In summary:

$f^{-1}(x) = \sqrt{x} - 1$

Domain of $f^{-1}(x)$: $x \geq 0$

2.

Range of g:

The range of g(x) can be determined by analyzing the linear expression $3x - 4$. Since the coefficient of x is positive, the function is increasing.

Also,

There are no restrictions on x other than $x \geq 2$. Therefore, the range of g(x) is all numbers for $x \geq 2$:

Inverse Function $g^{-1}(x)$ and its Domain:

To find the inverse function $g^{-1}(x)$, interchange x and y in the equation $g(x) = 3x – 4$ and solve for y:

$$x = 3y - 4$$

Solve for y:

$$y = \frac{x + 4}{3}$$

So, the inverse function $g^{-1}(x)$ is given by $g^{-1}(x) = = \frac{x+4}{3}$ is defined for all real numbers.

Therefore, the domain of $g^{-1}(x)$ is $\mathbb{R}$.

In summary:

$$g^{-1}(x) = \frac{x+4}{3}$$

Domain of $g^{-1}(x)$: $\mathbb{R}$

3)

Range of h:

The function $h(x) = e^x$ is the exponential function, and its range is all positive real numbers

Range of h = $\{y \epsilon \mathbb{R} | y > 0\}$

Inverse Function h-1(x) and its Domain:

To find the inverse function $h^{-1}(x)$, interchange x and y in the equation $h(x) = e^x$ and solve for y:

$$x = e^y$$

Taking the natural logarithm of both sides:

$$In(x) = y$$

So, the inverse function $h^{-1}(x)$ is given by $h^{-1}(x) = In(x)$.

Now, let's consider the domain of $h^{-1}(x)$. The natural logarithm is defined only for positive real numbers, so the domain of $h^{-1}(x)$ is $x > 0$.

In summary

$$h^{-1}(x) = In(x)$$

Domain of $^{h-1}(x)$: $x > 0$

4)

Range of p:

To find the range of p(x), we can complete the square on the quadratic expression:

$$p(x) = x^2 – 4x + 3$$

$$= (x – 2)^2 – 1$$

The square of any real number is non – negative, so (x -2)2 is always greater than or equal to zero. Therefore, the range of p(x) is all real numbers except when (x – 2)2 = 0, which occurs when x = 2. So, the range of p is all real numbers except – 1:

Range of p = $\{y \epsilon \mathbb{R} \mid y \neq -1\}$

Inverse Function $p^{-1}(x)$ and its Domain:

To find the inverse function $p^{-1}(x)$, interchange x and y in the equation

$$p(x) = (x -2)^2 - 1$$

and solve for y:

$$x = (y – 2)^2 – 1$$

Adding 1 to both sides:

$$x + 1 = (y – 2)^2$$

Taking the square root of both sides:

$$\sqrt{x+1}\ y- 2$$

Solve for y:

$$Y = \sqrt{x+1} + 2$$

So, the inverse function p-1(x) is given by p-1(x) = $\sqrt{x+1}$ + 2.

Now, let’s consider the domain of p-1(x). The square root is defined only for non – negative real numbers, so $\sqrt{x+1}$ is defined when x + 1 ≥ 0.

Therefore, the domain of p-1(x) is x ≥ -1.

In summary:

p-1(x) = $\sqrt{x+1}$ + 2

Domain of p-1(x): x ≥ - 1

5)

Ranger of p:

The function p(x) = $\sqrt{x+3}$ is a square root function with a shift of 3 unit to the left. The square root a defined only for non – negative values, and the shift ensures that the function is always, defined for x ≥ - 3. As x varies over all real numbers greater than or equal to – 3, the expression $\sqrt{x+3}$ covers all non – negative real numbers. So, the range of p(x) is $\{y \epsilon \mathbb{R} \,|y \geq 0\}$.

Inverse Function p-1(x) and its Domain:

To find the inverse function $p^{-1}(x)$, interchange x and y in the equation p(x) = $\sqrt{x+3}$ and solve for y:

x= $\sqrt{y+3}$

Squaring both sides:

$x^2 = y + 3$

Solving for y:

$y = x^2 – 3$

So, the inverse function $p^{-1}(x)$ is given by:

$p^{-1}(x) = x^2 – 3$

Now, let's consider the domain of p-1(x). The inverse function is defined for all real numbers, so the domain of $p^{-1}(x)$ is ℝ.

In summary:

$p^{-1}(x) = x^2 – 3$

Domain of $p^{-1}(x)$: ℝ

6)

Range of y:

The function $q(x) = 2^x$ is an exponential function with a base of 2. As x varies over all real numbers, the expression 2^x covers all positive real numbers.

So, the range of q(x) is $\{y \epsilon \mathbb{R} \,|y > 0\}$.

Inverse Function $q^{-1}(x)$ and its Domain:

To find the Inverse Function $q^{-1}(x)$, interchange x and y in the equation q(x) = 2x and solve for y:

$x = 2^y$

Taking the logarithm base 2 of both sides:

$\log_2(x) = y$

So, the inverse function $y^{-1}(x)$ is given by:

$q^{-1}(x) = \log_2(x)$

Now, let's consider the domain of y-1(x). The logarithmic function only for positive real numbers, so the domain of $q^{-1}(x)$ is $\{x \in \mathbb{R} \mid x > 0\}$.

In summary:

$q^{-1}(x) = \log_2(x)$

Domain of $q^{-1}(x)$: $\{x \in \mathbb{R} \mid x > 0\}$.

7)

Range of r:

The function $r(x) = (x - 1)^3$ is a cubic function. As x varies over all real numbers, the expression (x – 1)3 can take on all real values.

So, the range of r(x) is $\{y \in \mathbb{R}\}$.

Inverse Function r-1(x) and its Domain:

To find the inverse function $r^{-1}(x)$, interchange x and y In the equation $r(x) = (x - 1)^3$ and solve for y:

$x = (y - 1)^3$

Taking the cube root of both sides:

$y - 1 = \sqrt[3]{x}$

Solving for y:

$y = \sqrt[3]{x} + 1$

So, the inverse function $r^{-1}(x)$ is given by:

$r^{-1}(x) = \sqrt[3]{x} + 1$

Now, let's consider the domain of $r^{-1}(x)$. The cube root function is defied for all real numbers. So, the domain of $r^{-1}(x)$ is ℝ.

In summary:

$r^{-1}(x) = \sqrt[3]{x} + 1$

Domain of r-1(x): ℝ

8)

Range of s:

The function s(x) = $-\lceil r + 2 \rceil$ is an absolute value function with a reflection, and the negative sign ensures that the output is always negative. As x varies over all real numbers, the expression = $-\lceil r + 2 \rceil$ covers all real numbers but is always negative. So, the range of s(x) is $\{x \epsilon \mathbb{R} \,|x < 0\}$.

Inverse Function $s^{-1}(x)$ and its Domain:

To find the inverse function $s^{-1}(x)$, interchange x and y in the equation

s(x) = $-\lceil r + 2 \rceil$ and solve for y:

x = $-\lceil r + 2 \rceil$

Dividing both sides by – 1 to isolate the absolute value:

- x = $\lceil r + 2 \rceil$

Now, consider two cases:

1. x ≥ 0: - x = y + 2, so y = - x – 2.
2. x < 0: - x = - (y + 2), so y = - x – 2.

   So, the inverse function $s^{-1}(x)$ is given by:

   $s^{-1}(x) = - x – 2$

Now, let's consider the domain of $s^{-1}(x)$. The inverse function is defined for all real numbers, so the domain of $s^{-1}(x)$ is ℝ.

in summary:

$s^{-1}(x) = - x – 2$

Domain of $x^{-1}(x)$: ℝ

DR. ASHAR RANA

M.B.B.S / MS. CHEMISTRY

CHEMISTRY ONLINE
— TUITION —

Phone: +442081445350
www.chemistryonlinetuition.com
Email: asherrana@chemistryonlinetuition.com

- Founder & CEO of Chemistry Online Tuition Ltd.
- Completed Medicine (M.B.B.S) in 2007
- Tutoring students in UK and worldwide since 2008
- CIE & EDEXCEL Examiner since 2015
- Chemistry, Physics, Math’s and Biology Tutor

CONTACT INFORMATION FOR **CHEMISTRY ONLINE TUITION**

- UK Contact: 02081445350
- International Phone/WhatsApp: 00442081445350
- Website: www.chemistryonlinetuition.com
- Email: asherrana@chemistryonlinetuition.com

Address: 210-Old Brompton Road, London SW5 OBS, UK