Phone: +442081445350

www.chemistryonlinetuition.com

Email:asherrana@chemistryonlinetuition.com

# PURE MATH

## ALGEBRA AND FUNCTION

| | |
|---|---|
| Level & Board | EDEXCEL (A-LEVEL) |
| TOPIC: | FUNCTIONS |
| PAPER TYPE: | SOLUTION - 4 |
| TOTAL QUESTIONS | 8 |
| TOTAL MARKS | 38 |

**1)**

**Range of h:**

The function $h(x) = 2^{x-1}$ is an exponential function with a base of 2. As x varies over all

real numbers, the exponential function h(x) takes on all positive values. So, the range

of h is $\{y \epsilon \mathbb{R} \,|y > 0\}$.

**Inverse Function h-1(x) and its Domain:**

To find the inverse function h-1(x), interchange x and y in the equation $h(x) – 2^{x-1}$ and solve for y:

$x = 2^{y-1}$

Taking the logarithm base 2 of both sides:

$\log_2(x) = y – 1$

Solving for y:

$y = \log_2(x) + 1$

So, the inverse function $h^{-1}(x)$ is given by:

$h\text{-}1(x) = \log_2(x) + 1$

Now, let's consider the domain of $h^{-1}(x)$. The logarithmic function is defined only for positive real numbers, so the domain of $h^{-1}(x)$ is $\{x \epsilon \mathbb{R} \,|x > 0\}$.

In summary:

$h^{-1}(x) = \log_2(x) + 1$

Domain of $h^{-1}(x)$: $\{x \epsilon \mathbb{R} \,|x > 0\}$.

**2)**

**Range of p:**

The quadratic function p(x) = (x – 3)2 + 4 has its veretex at (3,1),, and the square term (x – 3)2 is always non – negative. Therefore, the minimum value of p(x) is 4, and as x varies over all real numbers, p(x) takes on all values greater than or equal to 4. So, the range of p is

$\{x \in \mathbb{R} \mid x \geq 4\}$.

Inverse Function $p^{-1}(x)$ and its Domain:

To find the inverse function $p^{-1}(x)$, interchange x and y in the equation p(x)

$= (x - 3)^2 + 4$

and solve for y:

$$y = \pm\sqrt{x - 4} + 3$$

Since p(x) is an upward – opening parabola, only the positive square root is considered to keep it as a function. So, the inverse function $p^{-1}(x)$ is given by:

$p^{-1}(x) = \sqrt{x - 4} + 3$

Now, let's consider the domain or p-1(x). The square root function is defined only for non – negative real numbers, so the domain of $p^{-1}(x)$ is $\{x \in \mathbb{R} \mid x \geq 4\}$.

In summary:

$p^{-1}(x) = \sqrt{x - 4} + 3$

Domain of $p^{-1}(x)$: $\{x \in \mathbb{R} \mid x \geq 4\}$

**3)**

**Range of m:**

The cubic function m(x) = x3 – 4x is defined for all real numbers. To find the range, we can analyze its behavior. The leading term is x3, which means

the function increases without bound as x approaches positive or negative infinity. Therefore, the range of m(x) is all real numbers: Range of m = $\{y \in \mathbb{R}\}$

**Inverse Function $m^{-1}(x)$ and its Domain:**

Finding the inverse function for a cubic function can be challenging due to its non – one – to – one nature. Cubic functions are not always invertible, and in this case, we may not able to find a simple algebraic expression for $m^{-1}(x)$.

In case where the inverse functions is not easily expressed algebraically, it may be represented graphically or numerically.

If we attempt to find the inverse function, we swap x and y and try to solve for y:

$x = y^3 - 4y$

However, solving this cubic equation for y is generally complex and may involve numerical methods.

So, for this example, we may not provide a simple algebraic expression for the inverse function. The focus would be on the cubic function itself and its behavior.

In summary:

Range of m = $\{y \in \mathbb{R}\}$

The inverse function m-1(x) may not have a simple algebraic expression.

**4)**

**Range of n:**

The function n(x) = $\frac{2x}{x-4}$ is a rational function. As x approaches 4, the denominator

(x -4) approaches O, leading to an undefined value. Therefore, the function has a vertical asymptote at x = 4. Apart from that, the function can take any real value.

So, the range of n(x) is $\{x \in \mathbb{R} \mid x \neq 0\}$ since the function is undefined when x = 4.

**Inverse Function $n^{-1}(x)$ and its Domain:**

To find the inverse function n-1(x). interchange x and y in the equation n(x) = $\frac{2x}{x-4}$ and solve for y:

(x) = $\frac{2x}{x-4}$

Now, solving for y:

y = $\frac{4x}{x-2}$

So, the inverse function $n^{-1}(x)$ is given by:

n-1(x) = $\frac{4x}{x-2}$

So, the inverse function $n^{-1}(x)$ is given by:

$n^{-1}(x)$ = $\frac{4x}{x-2}$

Now, let's consider the domain of $n^{-1}(x)$. The inverse function is defined for all real numbers except where the original function is undefined. Since n(x) undefined at x = 4, the domain of $n^{-1}(x)$ is ℝ, excluding x = -2 (where the denominator (x + 2) is equal to 0).

In summary:

n-1(x) = $\frac{4x}{x-2}$

Domain of n-1(x): ℝ, excluding x = - 2.

**5.**

Range of r:

The logarithmic function log2(x + 4) is defined for x > -4. The range of r(x) will be all

real numbers because the logarithm is defined for positive real numbers.

Range of r = $\{y \in \mathbb{R}\}$

Inverse Function $r^{-1}(x)$ and its Domain:

To find the inverse function $x^{-1}(x)$, interchange x and y in the equation

$r(x) = \log_2(x + 4)$

and solve for y:

$x = \log_2(y + 4)$

rewrite in exponential form:

$2^x = y + 4$

Solve for y:

$Y = 2^x - 4$

So, the inverse function $r^{-1}(x)$ is given by $r^{-1}(x) = 2^x – 4$.

Now, let's consider the domain of r-1(x). The exponential function $2^x$ is defined for all real numbers, and subtracting 4 does not impose any additional restrictions.

Therefore, the domain of $r^{-1}(x)$ is $\mathbb{R}$.

**6.**

Range of t:

The cubic function $t(x) = x^3 + 2x^2 – x – 3$ is defined for all real numbers. To find the

range, we can analyze its behavior. The leading term is $x^3$, which means the function increases without bound as x approaches positive or negative infinity. Therefore, the range of t(x) is all real numbers:

Range of t = $\{y \in \mathbb{R}\}$

Inverse Function $t^{-1}(x)$ and its Domain:

Finding the inverse function for a cubic function can be challenging due to its non – one – to – one nature. Cubic functions are not always invertible, and in this case, we may not be able to find a simple algebraic expression for $t^{-1}(x)$.

In cases where the inverse function is not easily expressed algebraically, it may be represented graphically or numerically.

If we attempt to find the inverse function, we swap x and y and try to solve for y:

$$x = y^3 + 2y^2 – y – 3$$

However, solving this cubic equation for y is generally complex may involve numerical method.

However, solving this cubic equation for y is generally complex and may involve numerical method.

So, for this example, we may not provide a simple algebraic expression for the inverse function. The focus would be on the cubic function itself and its behavior.

In summary:

Range of t = $\{y \in \mathbb{R}\}$

The inverse function $t^{-1}(x)$ may not have a simple algebraic expression.

**7.**

Range of $f$:

The function $v(x) = \log_3(|x| + 2)$ involves a logarithm with a base of 3.

The argument of the logarithm is $|x| + 2$, and $|x| + 2$ is always greater than or equal to 2.

The logarithm is defined for positive arguments, so the minimum value of $f(x)$ is $\log_3(2)$,

which is greater than O. therefore, the range of $f(x)$ is all real numbers, excluding negative values.

so, the range of $f$ is $\{y \in \mathbb{R} | \ y > 0\}$

**8.**

Range of g:

The function g(x) = $\frac{1}{x-1}$ is defined for all real numbers except x – 1. As x approaches 1 from the left (x → $1^-$), g(x) goes to negative infinity, and as x approaches 1 from the right (x → $1^-$), g(x) goes to positive infinity. Therefore, the range of g(x) is all real numbers, excluding O.

So, the range of g is $\{y \epsilon \mathbb{R} | \ y \neq 0 \}$

Inverse Function g-1(x) and its Domain:

To find the inverse function g-1(x), interchange x and y in the equation

$$g(x) = \frac{1}{x-1}$$

for y:

$$x = \frac{1}{x-1}$$

solving for y:

$$y = \frac{1}{x} + 1$$

so the inverse function $g^{-1}(x)$ is given by

$$g^{-1}(x) = \frac{1}{x} + 1$$

Now, let's consider the domain of g-1(x). The inverse function is defined for all real numbers except x - 0 (since division by zero is undefined).

In summary:

$g^{-1}(x) = \frac{1}{x} + 1$

Domain of $g^{-1}(x)$: $\mathbb{R}$, exchanging x - 0

www.chemistryonlinetuition.com asherrana@chemistryonlinetuition.com

DR. ASHAR RANA
M.B.B.S / MS. CHEMISTRY

CHEMISTRY ONLINE TUITION
Phone: +442081445350
www.chemistryonlinetuition.com
Email: asherrana@chemistryonlinetuition.com

- Founder & CEO of Chemistry Online Tuition Ltd.
- Completed Medicine (M.B.B.S) in 2007
- Tutoring students in UK and worldwide since 2008
- CIE & EDEXCEL Examiner since 2015
- Chemistry, Physics, Math's and Biology Tutor

CONTACT INFORMATION FOR
**CHEMISTRY ONLINE TUITION**

- UK Contact: 02081445350
- International Phone/WhatsApp: 00442081445350
- Website: www.chemistryonlinetuition.com
- Email: asherrana@chemistryonlinetuition.com

Address: 210-Old Brompton Road, London SW5 OBS, UK