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PURE MATH

ALGEBRA AND FUNCTION

Level & Board	EDEXCEL (A-LEVEL)
TOPIC:	FUNCTIONS
PAPER TYPE:	SOLUTION - 5
TOTAL QUESTIONS	8
TOTAL MARKS	38

3)

Range of h:

The function $h(x) = e^x$ is the exponential function, and its range is all positive real numbers

Range of h = $\{y \epsilon \mathbb{R} | y > 0\}$

Inverse Function h-1(x) and its Domain:

To find the inverse function $h^{-1}(x)$, interchange x and y in the equation $h(x) = e^x$ and solve for y:

$$x = e^y$$

Taking the natural logarithm of both sides:

$$In(x) = y$$

So, the inverse function $h^{-1}(x)$ is given by $h^{-1}(x) = In(x)$.

Now, let's consider the domain of $h^{-1}(x)$. The natural logarithm is defined only for positive real numbers, so the domain of $h^{-1}(x)$ is $x > 0$.

In summary

$$h^{-1}(x) = In(x)$$

Domain of $^{h-1}(x)$: $x > 0$

4)

Range of p:

To find the range of p(x), we can complete the square on the quadratic expression:

$$p(x) = x^2 – 4x + 3$$

$$= (x – 2)^2 – 1$$

The square of any real number is non – negative, so (x -2)2 is always greater than or equal to zero. Therefore, the range of p(x) is all real numbers except when (x – 2)2 = 0, which occurs when x = 2. So, the range of p is all real numbers except – 1:

Range of p = $\{y \in \mathbb{R} \mid y \neq -1\}$

Inverse Function $p^{-1}(x)$ and its Domain:

To find the inverse function $p^{-1}(x)$, interchange x and y in the equation

$$p(x) = (x - 2)^2 - 1$$

and solve for y:

$$x = (y - 2)^2 - 1$$

Adding 1 to both sides:

$$x + 1 = (y - 2)^2$$

Taking the square root of both sides:

$$\sqrt{x+1}\ y - 2$$

Solve for y:

$$Y = \sqrt{x+1} + 2$$

So, the inverse function p-1(x) is given by p-1(x) = $\sqrt{x+1}$ + 2.

Now, let's consider the domain of p-1(x). The square root is defined only for non – negative real numbers, so $\sqrt{x+1}$ is defined when $x + 1 \geq 0$.

Therefore, the domain of p-1(x) is $x \geq -1$.

In summary:

$$p\text{-}1(x) = \sqrt{x+1} + 2$$

Domain of p-1(x): $x \geq -1$

5.

Range of r:

The logarithmic function log2(x + 4) is defined for x > -4. The range of r(x) will be all real numbers because the logarithm is defined for positive real numbers.

Range of r = $\{y \epsilon \mathbb{R}\}$

Inverse Function $r^{-1}(x)$ and its Domain:

To find the inverse function $x^{-1}(x)$, interchange x and y in the equation

$r(x) = \log_2(x + 4)$

and solve for y:

$x = \log_2(y + 4)$

rewrite in exponential form:

$2^x = y + 4$

Solve for y:

$Y = 2^x - 4$

So, the inverse function $r^{-1}(x)$ is given by $r^{-1}(x) = 2^x – 4$.

Now, let’s consider the domain of r-1(x). The exponential function 2^x is defined for all real numbers, and subtracting 4 does not impose any additional restrictions.

Therefore, the domain of $r^{-1}(x)$ is $\mathbb{R}$.

6.

Range of t:

The cubic function $t(x) = x^3 + 2x^2 – x – 3$ is defined for all real numbers. To find the range, we can analyze its behavior. The leading term is x^3, which means the function increases without bound as x approaches positive or negative infinity. Therefore, the range of t(x) is all real numbers:

Range of t = $\{y \in \mathbb{R}\}$

Inverse Function $t^{-1}(x)$ and its Domain:

Finding the inverse function for a cubic function can be challenging due to its non – one – to – one nature. Cubic functions are not always invertible, and in this case, we may not be able to find a simple algebraic expression for $t^{-1}(x)$. in cases where the inverse function is not easily expressed algebraically, it may be represented graphically or numerically.

If we attempt to find the inverse function, we swap x and y and try to solve for y:

$$x = y^3 + 2y^2 - y - 3$$

However, solving this cubic equation for y is generally complex may involve numerical method.

However, solving this cubic equation for y is generally complex and may involve numerical method.

So, for this example, we may not provide a simple algebraic expression for the inverse function. The focus would be on the cubic function itself and its behavior.

In summary:

Range of t = $\{y \in \mathbb{R}\}$

The inverse function $t^{-1}(x)$ may not have a simple algebraic expression.

5)

Range of m:

The cubic function $m(x) = x^3 – 4x$ is defined for all real numbers. To find the range, we can analyze its behavior. The leading term is x3, which means the function increases without bound as x approaches positive or negative infinity. Therefore, the range of m(x) is all real numbers: Range of m = $\{y \epsilon \mathbb{R}\}$

Inverse Function $m^{-1}(x)$ and its Domain:

Finding the inverse function for a cubic function can be challenging due to its non – one – to – one nature. Cubic functions are not always invertible, and in this case, we may not able to find a simple algebraic expression for $m^{-1}(x)$.

In case where the inverse functions is not easily expressed algebraically, it may be represented graphically or numerically.

If we attempt to find the inverse function, we swap x and y and try to solve for y:

$x = y^3 – 4y$

However, solving this cubic equation for y is generally complex and may involve numerical methods.

So, for this example, we may not provide a simple algebraic expression for the inverse function. The focus would be on the cubic function itself and its behavior.

In summary:

Range of m = $\{y \epsilon \mathbb{R}\}$

The inverse function m-1(x) may not have a simple algebraic expression.

6)

Range of n:

The function n(x) = $\frac{2x}{x-4}$ is a rational function. As x approaches 4, the denominator

(x -4) approaches O, leading to an undefined value. Therefore, the function has a vertical asymptote at x = 4. Apart from that, the function can take any real value.

So, the range of n(x) is $\{x \epsilon \mathbb{R} \mid x \neq 0\}$ since the function is undefined when x = 4.

Inverse Function n^{-1}(x) and its Domain:

To find the inverse function n-1(x). interchange x and y in the equation n(x) = $\frac{2x}{x-4}$ and solve for y:

(x) = $\frac{2x}{x-4}$

Now, solving for y:

y = $\frac{4x}{x-2}$

So, the inverse function n^{-1}(x) is given by:

n-1(x) = $\frac{4x}{x-2}$

So, the inverse function n^{-1}(x) is given by:

$n^{-1}(x) = \frac{4x}{x-2}$

Now, let's consider the domain of n^{-1}(x). The inverse function is defined for all real numbers except where the original function is undefined. Since n(x) undefined at x = 4, the domain of n^{-1}(x) is ℝ, excluding x = -2 (where the denominator (x + 2) is equal to 0).

In summary:

n-1(x) = $\frac{4x}{x-2}$

Domain of n-1(x): ℝ, excluding x = - 2.

7)

Ranger of p:

The function p(x) = $\sqrt{x+3}$ is a square root function with a shift of 3 unit to the left.

The square root a defined only for non – negative values, and the shift ensures that

the function is always, defined for x ≥ - 3. As x varies over all real numbers greater than or equal to – 3, the expression $\sqrt{x+3}$ covers all non – negative real numbers.

So, the range of p(x) is $\{y \in \mathbb{R} \mid y \geq 0\}$.

Inverse Function p-1(x) and its Domain:

To find the inverse function $p^{-1}(x)$, interchange x and y in the equation p(x)

= $\sqrt{x+3}$

and solve for y:

x= $\sqrt{y+3}$

Squaring both sides:

$x^2 = y + 3$

Solving for y:

$y = x^2 – 3$

So, the inverse function $p^{-1}(x)$ is given by:

$p^{-1}(x) = x^2 – 3$

Now, let’s consider the domain of p-1(x). The inverse function is defined for all real

numbers, so the domain of $p^{-1}(x)$ is ℝ.

In summary:

$p^{-1}(x) = x^2 – 3$

Domain of $p^{-1}(x)$: ℝ

8)

Range of y:

The function $q(x) = 2^x$ is an exponential function with a base of 2. As x varies over all real numbers, the expression 2^x covers all positive real numbers.

So, the range of q(x) is $\{y \in \mathbb{R} \,|y > 0\}$.

Inverse Function $q^{-1}(x)$ and its Domain:

To find the Inverse Function $q^{-1}(x)$, interchange x and y in the equation

q(x) = 2x and

solve for y:

$x = 2^y$

Taking the logarithm base 2 of both sides:

$\log_2(x) = y$

So, the inverse function $y^{-1}(x)$ is given by:

$q^{-1}(x) = \log_2(x)$

Now, let’s consider the domain of y-1(x). The logarithmic function only for positive

real numbers, so the domain of $q^{-1}(x)$ is $\{x \in \mathbb{R} \,|x > 0\}$.

In summary:

$q^{-1}(x) = \log_2(x)$

Domain of $q^{-1}(x)$: $\{x \in \mathbb{R} \mid x > 0\}$.