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PURE MATH

ALGEBRA AND FUNCTION

Level & Board	EDEXCEL (A-LEVEL)
TOPIC:	FUNCTIONS
PAPER TYPE:	SOLUTION - 6
TOTAL QUESTIONS	8
TOTAL MARKS	38

1.

Range of f:

The function v(x) = $\log_3(|x| + 2)$ involves a logarithm with a base of 3.

The argument of the logarithm is $|x|$ + 2, and $|x|$ + 2 is always greater than or equal to 2.

The logarithm is defined for positive arguments, so the minimum value of f(x) is $\log_3(2)$,

which is greater than O. therefore, the range of f(x) is all real numbers, excluding negative values.

so, the range of f is $\{y \epsilon \mathbb{R} | \ y > 0\}$

2.

Range of g:

The function g(x) = $\frac{1}{x-1}$ is defined for all real numbers except x – 1. As x approaches 1 from the left (x → 1^-), g(x) goes to negative infinity, and as x approaches 1 from the right (x → 1^-), g(x) goes to positive infinity. Therefore, the range of g(x) is all real numbers, excluding O.

So, the range of g is $\{y \epsilon \mathbb{R} | \ y \neq 0 \}$

Inverse Function g-1(x) and its Domain:

To find the inverse function g-1(x), interchange x and y in the equation g(x) - $\frac{1}{x-1}$ for y:

$$x = \frac{1}{x-1}$$

solving for y:

$$y = \frac{1}{x} + 1$$

so the inverse function g^{-1}(x) is given by

$$g^{-1}(x) = \frac{1}{x} + 1$$

Now, let's consider the domain of g-1(x). The inverse function is defined for all real numbers except x - 0 (since division by zero is undefined).

In summary:

$g^{-1}(x) = \frac{1}{x} + 1$

Domain of $g^{-1}(x)$: $\mathbb{R}$, exchanging x - 0

3.

Range of f:

To find the range of f(x), we can complete the square on the quadratic expression:

$$f(x) = x^2 + 2x + 1$$
$$= (x + 1)^2$$

The square of any real number is non – negative, so (x +1)2 is always greater than or equal to zero. Therefore, the range of f(x) is all non – negative real numbers:

$$\text{Range of } f = \{y \, \epsilon \, \mathbb{R} \mid y \ \geq 0\}$$

Inverse Function f^{-1}(x) and its Domain:

To find the inverse function f^{-1}(x), interchange x and y in the equation $f(x) = (x+1)^2$ and solve for y:

Taking the square root of both sides:

$$\sqrt{x} = y = 1$$

Solving for y:

$$y = \sqrt{x} - 1$$

So, the inverse function f^{-1}(x) is given by $f^{-1}(x) = \sqrt{x} - 1$.

Now, let's consider the domain of f^{-1}(x). The square root is defined only for non – negative real numbers, so $\sqrt{x}$ is defined when $x \geq 0$. Also, subtracting 1 does not affect this domain restriction. Therefore, the domain of f^{-1}(x) is $x \geq 0$.

In summary:

$f^{-1}(x) = \sqrt{x} - 1$

Domain of $f^{-1}(x)$: $x \geq 0$

4.

Range of g:

The range of g(x) can be determined by analyzing the linear expression 3x – 4. Since the coefficient of x is positive, the function is increasing.

Also,

There are no restrictions on x other than x ≥ 2. Therefore, the range of g(x) is all numbers for x ≥ 2:

Inverse Function $g^{-1}(x)$ and its Domain:

To find the inverse function $g^{-1}(x)$, interchange x and y in the equation g(x) = 3x – 4 and solve for y:

x = 3y -4

Solve for y:

$$y = \frac{x+4}{3}$$

So, the inverse function $g^{-1}(x)$ is given by $g^{-1}(x) = = \frac{x+4}{3}$ is defined for all real numbers.

Therefore, the domain of $g^{-1}(x)$ is $\mathbb{R}$.

In summary:

$g^{-1}(x) = \frac{x+4}{3}$

Domain of $g^{-1}(x)$: $\mathbb{R}$

5)

Range of h:

The function $h(x) = 2^{x-1}$ is an exponential function with a base of 2. As x varies over all real numbers, the exponential function h(x) takes on all positive values. So, the range

of h is $\{y \in \mathbb{R} \mid y > 0\}$.

Inverse Function h-1(x) and its Domain:

To find the inverse function h-1(x), interchange x and y in the equation h(x) – 2^{x-1} and solve for y:

$x = 2^{y-1}$

Taking the logarithm base 2 of both sides:

$\log_2(x) = y - 1$

Solving for y:

$y = \log_2(x) + 1$

So, the inverse function $h^{-1}(x)$ is given by:

h-1(x) = $\log_2(x) + 1$

Now, let's consider the domain of $h^{-1}(x)$. The logarithmic function is defined only for positive real numbers, so the domain of $h^{-1}(x)$ is $\{x \epsilon \mathbb{R} \,|x > 0\}$.

In summary:

$h^{-1}(x) = \log_2(x) + 1$

Domain of $h^{-1}(x)$: $\{x \epsilon \mathbb{R} \,|x > 0\}$.

6)

Range of p:

The quadratic function p(x) = (x – 3)2 + 4 has its veretex at (3,1),, and the square term (x – 3)2 is always non – negative. Therefore, the minimum value of p(x) is 4, and as x varies over all real numbers, p(x) takes on all values greater than or equal to 4. So, the range of p is

$\{x \epsilon \mathbb{R} \,|x \geq 4\}$.

Inverse Function $p^{-1}(x)$ and its Domain:

To find the inverse function $p^{-1}(x)$, interchange x and y in the equation p(x)

$= (x - 3)^2 + 4$

and solve for y:

$$y = \pm\sqrt{x - 4} + 3$$

Since p(x) is an upward – opening parabola, only the positive square root is considered to keep it as a function. So, the inverse function $p^{-1}(x)$ is given by:

$p^{-1}(x) = \sqrt{x - 4} + 3$

Now, let's consider the domain or p-1(x). The square root function is defined only for non – negative real numbers, so the domain of $p^{-1}(x)$ is $\{x \epsilon \mathbb{R} \,|x \geq 4\}$.

In summary:

$p^{-1}(x) = \sqrt{x-4} + 3$

Domain of $p^{-1}(x)$: $\{x \epsilon \mathbb{R} \mid x \geq 4\}$

7)

Range of r:

The function $r(x) = (x - 1)^3$ is a cubic function. As x varies over all real numbers, the expression (x – 1)3 can take on all real values.

So, the range of r(x) is $\{y \epsilon \mathbb{R}\}$.

Inverse Function r-1(x) and its Domain:

To find the inverse function $r^{-1}(x)$, interchange x and y In the equation $r(x) = (x - 1)^3$ and solve for y:

$x = (y - 1)^3$

Taking the cube root of both sides:

$y - 1 = \sqrt[3]{x}$

Solving for y:

$y = \sqrt[3]{x} + 1$

So, the inverse function $r^{-1}(x)$ is given by:

$r^{-1}(x) = \sqrt[3]{x} + 1$

Now, let's consider the domain of $r^{-1}(x)$. The cube root function is defied for all real numbers. So, the domain of $r^{-1}(x)$ is $\mathbb{R}$.

In summary:

$r^{-1}(x) = \sqrt[3]{x} + 1$

Domain of r-1(x): $\mathbb{R}$

8)

Range of s:

The function s(x) = $-\lceil r + 2 \rceil$ is an absolute value function with a reflection, and the negative sign ensures that the output is always negative. As x varies over all real numbers, the expression = $-\lceil r + 2 \rceil$ covers all real numbers but is always negative. So, the range of s(x) is $\{x \in \mathbb{R} \mid x < 0\}$.

Inverse Function s^{-1}(x) and its Domain:

To find the inverse function s^{-1}(x), interchange x and y in the equation

s(x) $= -\lceil r + 2 \rceil$ and solve for y:

x = $-\lceil r + 2 \rceil$

Dividing both sides by – 1 to isolate the absolute value:

- x $= \lceil r + 2 \rceil$

Now, consider two cases:

1. x ≥ 0: - x = y + 2, so y = - x – 2.
2. x < 0: - x = - (y + 2), so y = - x – 2.

   So, the inverse function s^{-1}(x) is given by:

   s^{-1}(x) = - x – 2

Now, let's consider the domain of s^{-1}(x). The inverse function is defined for all real numbers, so the domain of s^{-1}(x) is $\mathbb{R}$.

in summary:

s^{-1}(x) = - x – 2

Domain of x^{-1}(x): $\mathbb{R}$

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