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## PURE MATH

## ALGEBRA AND FUNCTION

Q. 1

- (i) $3 x^{3}-17 x^{2}-6 x=0$

We can first factor out the common form of $x$
$\Rightarrow \quad x\left(3 x^{2}-17 x-6\right)=0$
$\Rightarrow \quad x=0 \quad$ or $\quad 3 x^{2}-17 x-6=0$
Now,

$$
3 x^{2}-17 x-6=0
$$

Here

$$
a=3, b=-17, c=-6
$$

Using quadratic formula

$$
x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}
$$

Plugging these values into the formula

$$
\begin{array}{ll}
x=\frac{-(-17) \pm \sqrt{\left.(-17)^{2}-4\right)(3)(-6)}}{2(3)} \\
x=\frac{17 \pm \sqrt{284+72}}{6} & \\
x=\frac{17 \pm \sqrt{361}}{6} & \\
x=\frac{17 \pm 19}{6} & \text { or } \\
x=\frac{17+19}{6} & x=\frac{17-19}{6} \\
x=\frac{36}{6} & x=\frac{-2}{6} \\
x=6 & ,
\end{array}
$$

- Now, We have three solution for the equation

$$
\begin{aligned}
& 3 x^{3}-17 x^{2}-6 x=0 \\
& x=0, x=6, x=\frac{-1}{3}
\end{aligned}
$$

so, these are all the solutions to the given condition.

- (ii) $3(y-2)^{6}-17(y-2)^{4}-6(y-2)^{2}=0$

$$
\begin{aligned}
& \text { Let } \quad(y-2)^{2}=x \\
& \Rightarrow \quad 3 x^{3}-17 x^{2}-6 x=0 \\
& \Rightarrow \quad x\left(3 x^{2}-17 x-6\right)=0 \\
& x=0 \quad \text { or } \quad 3 x^{2}-17 x-6=0 \\
& \text { But , Here } \\
& (y-2)^{2}=0 \quad, \quad a=3, b=-17, c=-6 \\
& y-2=0, \quad \text { using quadratic formula } \\
& y=2, \quad x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a} \\
& x=\frac{-(-17) \pm \sqrt{(17)^{2}-4(3)(-6)}}{2(3)} \\
& \text {, } x=\frac{17 \pm \sqrt{36}}{6} \\
& \text {, } x=\frac{17 \pm 19}{6} \\
& \text {, } \quad x=\frac{17+19}{6}, x=\frac{17-19}{6} \\
& \text {, } x=6 \quad, x=\frac{-1}{3} \\
& x=6 \quad, \quad x=\frac{-1}{3} \\
& \Rightarrow \quad(y-2)^{2}=6, \quad(y-2)^{2}=\frac{-1}{2} \\
& \Rightarrow y-2= \pm \sqrt{6} \text {, However, since the square of a real }
\end{aligned}
$$

number

$$
y-2= \pm \sqrt{6} \text {, cannot be negative a this equation }
$$

has no

$$
\begin{aligned}
& y-2= \pm \sqrt{6} \text {, real solution for } y \text {. } \\
& \text { and } \\
& y=2 \pm \sqrt{6}
\end{aligned}
$$

So, the real solutions for the original equation

$$
3(y-2)^{6}-17(y-2)^{4}-6(y-2)^{2}=0
$$

are:
$y=2+\sqrt{6} \quad y=2-\sqrt{6} \quad y=2$

## Q. 2

The discriminant is given by:
Discriminant $=0$

$$
b^{2}-4 a c=0
$$

But, the coefficient are

$$
a=k, b=k-3, c=1
$$

$\Rightarrow$
$(k-3)^{2}-4(k)(1)=0$
$k^{2}+9-6 k-4 k=0$
$k^{2}-10 k+9=0$
Factorization

$$
\begin{aligned}
& k^{2}-9 k-k+9=0 \\
& k(k-9)-1(k-9)=0 \\
& (k-1)(k-9)=0 \\
& k-1=0, \quad k-9=0 \\
& k=1, \quad k=9
\end{aligned}
$$

so, the possible values of $k$ for the quadratic equation to have two equal real roots are:
$k=9$ and $k=1$.
Q. 3
(a)

$$
\begin{aligned}
& \text { substitute } x=-4 \text { into } k x^{2}+(3 k+1) x-8=0 \\
& k(-4)^{2}+(3 k+1)(-4)-8=0 \\
& \Rightarrow \\
& \quad 16 k-12 k-4-8=0 \\
& 4 k-12=0 \\
& 4 k=12 \\
& k=3
\end{aligned}
$$

(b)

First we have the value of $k$, which is 4 , so $k=3$
$\therefore$ By parts (a)
$\Rightarrow$

$$
\begin{aligned}
& k x^{2}+(3 k+1) x+8=0 \\
& 3 x^{2}+10 x-8=0
\end{aligned}
$$

Factorization

$$
\begin{aligned}
& 3 x^{2}+12 x-2 x-8=0 \\
& 3 x(x+4)-2(x+4)=0 \\
& (3 x-2)(x+4)=0 \\
& 3 x-2=0 \text { or } x+4=0 \\
& x=\frac{2}{3} \quad \text { or } x=-4
\end{aligned}
$$

so, the second possible value of $x$ is:

$$
x=\frac{2}{3} \quad \text { and } x=-4
$$

Q. 4

This equation is Based on the nature of roots.
Let
Equation is $a x^{2}+b x+c=0$ and the discriminant $=b^{2}-4 a c$
If Discriminant $=0$
Then equation has real roots.
$b^{2}-4 a c=0$
Here, $a=k+3, b=2(k+3), c=4$
$\Rightarrow \quad[2(k+3)]^{2}-4[k+3][4]=0$
$\Rightarrow \quad 4(k+3)^{2}-16(k+3)=0$
Dividing by 4 on both side
$(k+3)^{2}-4(k+3)=0$
$k^{2}+6 k+9-4 k-12=0$
$k^{2}+2 k-3=0 \quad \because$ Factorize
$k^{2}+3 k-k-3=0$
$k(k+3)-1(k+3)=0$
$(k+3)(k-1)=0$
$k=-3, k=1$
But $k=-3$ not possible (Coefficient of $x^{2} \pm 0$
so, value of $k=1$.
Q. 5

The quadratic equation is given
$x^{2}-4 x-1=2 p(x-5)$
$x^{2}-4 x-1=2 p x-10 p$
$x^{2}-4 x-2 p x-1+10 p=0$

$$
\begin{array}{ll}
\Rightarrow & x^{2}+(-4-2 p) x+(10 p-1)=0 \\
& \text { Here, } \\
& a-1, b=-4-2 p, c=10 p-1 \\
& \text { Two equal root } \\
& b^{2}-4 a c=0 \\
& (-4-2 p)^{2}-4(1)(10 p-1)=0 \\
& 16+16 p+4 p^{2}-40 p+4=0 \\
& 4 p^{2}-24 p+20=0 \\
\Rightarrow \quad p^{2}-6 p+5=0 \\
& \text { Factorization } \\
& p^{2}-5 p-p+5=0 \\
& p(p-5)-1(p-5)=0 \\
& (p-1)(p-5)=0 \\
& p-1=0 \text { or } p-5=0 \\
& p=1 \\
\text { So, } \\
& P=1,5
\end{array}
$$

Q. 6

Given

$$
2 q x^{2}+q x-1=0
$$

For a quadratic equation,

$$
a x^{2}+b x+c=0
$$

The expression for solutions

$$
x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}
$$

Where $b^{2}-4 a c$ is called discriminant.
(a) Since, given is a quadratic equation with no real roots, its discriminant must be:

$$
\begin{aligned}
& b^{2}-4 a c<0 \\
& (q)^{2}-4(2 q)(-1)<0 \\
& q^{2}+8 q<0 \quad \text { (Hence Proved) }
\end{aligned}
$$

(b)

$$
\begin{aligned}
& q^{2}+8 q=0 \\
& q(q+8)=0 \\
& q=0 \quad \text { or } \quad q=-8
\end{aligned}
$$

so, the critical points on curve for given condition are -8 and
0.

$$
\begin{aligned}
& \text { Therefore, conditions for } \\
& \begin{array}{l}
q^{2}+8 q<0 \quad \text { are: } \\
q>-8 \\
q<0 \\
\Rightarrow \quad \\
-8<q<0
\end{array}
\end{aligned}
$$

Q. 7
$4-3 x-x^{2}$
$4-\left(x^{2}+3 x\right)$
Complete the square coefficient of the $x$ term: 3 divide it in half: $: \frac{3}{2}$ square it: $\left(\frac{3}{2}\right)^{2}$

$$
\begin{aligned}
& \text { Use }\left(\frac{3}{2}\right)^{2} \text { to complete the square: } \\
& =4+\left(\frac{3}{2}\right)^{2}-\left(x^{2}+3 x+\left(\frac{3}{2}\right)^{2}\right) \\
& =\frac{25}{4}-\left(x+\frac{3}{2}\right)^{2}
\end{aligned}
$$

## Q. 8

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