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PURE MATH

ALGEBRA AND FUNCTION

Level & Board	EDEXCEL (A-LEVEL)
TOPIC:	QUADRATICS
PAPER TYPE:	SOLUTION 1
TOTAL QUESTIONS	8
TOTAL MARKS	28

Quadratics I (solution)

Q.1

- To prove that the equation $Kx^2 + 4Kx + 3 = 0$ has no real roots if and only if $0 \leq K < \frac{3}{4}$, We can use the discriminant of a quadratic equation. The discriminant of a quadratic equation $ax^2 + bx + c = 0$ is given by $D = b^2 - 4ac$. For real roots, the discriminant must be greater than or equal to zero.

$$\Longrightarrow \quad kx^2 + 4kx + 3 = 0$$

Here

$$A = k \;, \; b = 4k \;, \; c = 3$$

Then,

$$D = b^2 - 4ac$$

$$D = (4k)^2 - 4(k)(3)$$

$$D = (16k)^2 - 12k$$

- To have real roots, D must be greater than or equal to zero.

$\Longrightarrow$

$$16k^2 - 12k \geq 0$$

$\Longrightarrow$

$$4k(4k - 3) \geq 0$$

Here,

- (1) 4k is always non-negative (zero or positive) for all real values of k.
- (2) $4k - 3$ is zero when $k = \frac{3}{4}$, and it's negative for $k < \frac{3}{4}$ and positive for $k > \frac{3}{4}$.

So, the inequality $4k(4k-3) \geq 0$ is true when

- Both factors are positive, which occurs when $k > \frac{3}{4}$.
- Both factors are positive, which occurs when $k = \frac{3}{4}$.

For real roots, we want the discriminant to be greater than or equal to zero, which corresponds to the values of k where $4k(4k-3) \geq 0$

Therefore, the equation $kx^2 + 4kx + 3 = 0$ has no real roots if and only if $k < \frac{3}{4}$. so, we have shown that $0 \leq k < \frac{3}{4}$ is the range of values for which the equation has no real roots.

Q.2

- (i) $3x^3 - 17x^2 - 6x = 0$

We can first factor out the common form of x

$$\Longrightarrow \quad x(3x^2 - 17x - 6) = 0$$

$\Longrightarrow \quad x = 0 \quad$ or $\quad 3x^2 - 17x - 6 = 0$

Now, $\quad 3x^2 - 17x - 6 = 0$

Here

$a = 3, \quad b = -17, c = -6$

Using quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Plugging these values into the formula

$$x = \frac{-(-17) \pm \sqrt{(-17)^2 - 4)(3)(-6)}}{2(3)}$$

$$x = \frac{17 \pm \sqrt{284 + 72}}{6}$$

$$x = \frac{17 \pm \sqrt{361}}{6}$$

$$x = \frac{17 \pm 19}{6}$$

$$x = \frac{17 + 19}{6} \quad \text{or} \quad x = \frac{17 - 19}{6}$$

$$x = \frac{36}{6} \quad , \quad x = \frac{-2}{6}$$

$$x = 6 \quad , \quad x = \frac{-1}{3}$$

- Now, We have three solution for the equation

  $3x^3 - 17x^2 - 6x = 0$

  $x = 0 , x = 6 , x = \frac{-1}{3}$

  So, these are all the solutions to the given condition.

- (ii) $3(y-2)^6 - 17(y-2)^4 - 6(y-2)^2 = 0$

  Let $\quad (y-2)^2 = x$

  $\Longrightarrow \quad 3x^3 - 17x^2 - 6x = 0$

  $\Longrightarrow \quad x(3x^2 - 17x - 6) = 0$

$x = 0 \quad$ or $\quad 3x^2 - 17x - 6 = 0$

But , Here

$(y-2)^2 = 0$, $a = 3 , b = -17 , c = -6$

$y - 2 = 0$, using quadratic formula

$y = 2$, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$, \quad x = \frac{-(-17) \pm \sqrt{(17)^2 - 4(3)(-6)}}{2(3)}$$

$$, \quad x = \frac{17 \pm \sqrt{36}}{6}$$

$$, \quad x = \frac{17 \pm 19}{6}$$

$$, \quad x = \frac{17+19}{6}, \; x = \frac{17-19}{6}$$

$$, \quad x = 6 \quad , x = \frac{-1}{3}$$

$X = 6 \quad , \quad x = \frac{-1}{3}$

$\Longrightarrow \quad (y-2)^2 = 6, \quad (y-2)^2 = \frac{-1}{2}$

$\Longrightarrow \quad Y - 2 = \pm\sqrt{6}$, However, since the square of a real number

$y - 2 = \pm\sqrt{6}$, cannot be negative a this equation has no

$y - 2 = \pm\sqrt{6}$, real solution for y.

and

$y = 2 \pm \sqrt{6}$

So, the real solutions for the original equation

$3(y-2)^6 - 17(y-2)^4 - 6(y-2)^2 = 0$

are:

$y = 2 + \sqrt{6} \quad Y = 2 - \sqrt{6} \quad Y = 2$

Q.3

(i) $16a^2 = 2\sqrt{a}$

Dividing by 2

$8a^2 = \sqrt{a}$

Squaring on both sides, we get

$64a^4 = a$

$\Longrightarrow \quad 64a^4 - a = 0$

$a(64a^3 - 1) = 0$

$\Longrightarrow$

$a = 0 \quad$ and $\quad 64a^3 - 1 = 0$

$\Longrightarrow \quad 64a^3 = 1$

$$\Longrightarrow \quad a^3 = \frac{1}{64}$$

$$\Longrightarrow \quad a^3 = \frac{1}{4^3}$$

$$\Longrightarrow \quad a = \frac{1}{4}$$

So,

The two real solutions to the equation $16a^2 = 2\sqrt{a}$ are $a = 0$ and $a = \frac{1}{4}$.

(ii) $\quad b^4 + 7b^2 - 18 = 0$

$\Longrightarrow \quad (b^2)^2 + 7(b^2) - 18 = 0$

Let $\quad y = b^2$

$\Longrightarrow \quad y^2 + 7y - 18 = 0$

$\Longrightarrow$ Factorization

$\Longrightarrow \quad y^2 + 9y - 2y - 18 = 0$

$\Longrightarrow \quad y(y+9) - 2(y+9) = 0$

$\Longrightarrow \quad (y-2)(y+9) = 0$

$\Longrightarrow \quad y - 2 = 0 \quad , \quad y + 9 = 0$

$\Longrightarrow \quad y = 2 \quad , \quad y = -9$

But

$b^2 = 2 \quad , \quad b^2 = -9$

$\Longrightarrow \quad b = \pm\sqrt{2}$

- For $b^2 = -9$, there are no real solutions because you cannot take the square root of a negative number.
- So, the real solutions to the equation $b^4 + 7b^2 - 18 = 0$ are $b = \sqrt{2}$ and $b = -\sqrt{2}$

Q.4

(a) $\quad 2x^2 + 4x + 9$

$= \left(x^2 + 2x + \frac{9}{2}\right)$

$= 2\left(x^2 + 2x + 1 - 1 + \frac{9}{2}\right)$

$= 2\left((x+1)^2 - 1 + \frac{9}{2}\right)$

$= 2\left((x+1)^2 \frac{7}{2}\right)$

$=2(x+1)^2 + 7$

(b)

At x = 0 , y = 9

Minimum at (-1 , 7)

(c)

f(x)= $2x^2$ + 4x + 9

Shift 2 right f(x-2)

$\Longrightarrow$

f(x-2) = $2(x-2)^2$+4(x-2) + 9

= $2(x-2)^2$+ 4x -8 + 9

= $2(x-2)^2$+ 4x + 1

Then to transform to g(x) need -4 full transformation is translate $\binom{2}{-4}$.

So by (ii),

f(x) = $2(x+1)^2 + 7$

Minimum point = (-1,7)

h(x) = $\frac{21}{2x^2+4x+9}$

h(x) = $21(2x^2+4x+9)^{-1}$

$\Longrightarrow$

h'(x) = -21 $(2x^2+4x+9)^{-2}$ (4x + 4)

h'(x) = $\frac{-21(4x+4)}{(2x^2+4x+9)^2}$

h'(x) = 0 when x = -1

h'(x) = $\frac{21}{2(-1)^2+4(-1)+9}$

= $\frac{21}{7}$ = 3

As

$x \to \infty$ or $x \to -\infty$

$\Longrightarrow$

h(x) $\to$ 0

Range 0< h(x)$\leq$3

Q.5

The discriminant is given by:

Discriminant = 0

$$b^2 - 4ac = 0$$

But, the coefficient are

$a = k$, $b = k-3$, $c = 1$

$\Longrightarrow$

$$(k-3)^2 - 4(k)(1) = 0$$

$$k^2 + 9 - 6k - 4k = 0$$

$$k^2 - 10k + 9 = 0$$

Factorization

$$k^2 - 9k - k + 9 = 0$$

$$K(k-9) - 1(k-9) = 0$$

$$(k-1)(k-9) = 0$$

$K - 1 = 0$, $k - 9 = 0$

$K = 1$, $k = 9$

So, the possible values of k for the quadratic equation to have two equal real roots are:

$k = 9$ and $k = 1$.

Q.6

(a) two real solutions

$\Longrightarrow$

$$b^2 - 4ac > 0$$

$$kx^2 + 4x + (5 - k+ = 0$$

$$(4)^2 - 4k\,(5 - k) > 0$$

$$16 - 20k + 4k^2 > = 0$$

$\Longrightarrow$

$$4k^2 - 20k + 16 > 0$$

$\Longrightarrow$

$$k^2 - 5k + 4 > 0$$

(b) As

$k^2 - 5k + 4 > 0$

$k^2 - k - 4k + 4 > 0$

$K(k-1) - 4(k-1) > 0$

$(k-4)(k-1) = 0$

$\Longrightarrow$

Either $k < 1$ or $k > 4$

Q.7

The discriminant is given by

Discriminant < 0

$b^2 - 4ac < 0$

$(6k)^2 - 4(k)(5) < 0$

$36k^2 - 20k < 0$

$4k(9k-5) < 0$

Divide both sides by 4

$K(9k-5) < 0$

Apply zero product property

$K = 0$ or $9k - 5 = 0$

, $K = \frac{5}{9}$

$\Longrightarrow$

$4k$ is positive, and $9k - 5$ is negative

$0 < k < \frac{5}{9}$

$\Longrightarrow$

$4k$ is negative, and $9k-5$ is positive

$0 > k > \frac{5}{9}$

$\Longrightarrow$

$K \in \left(0, \frac{5}{9}\right)$

Q.8

(a)

Substitute x = -4 into $kx^2 + (3k+1)x - 8 = 0$

$k(-4)^2 + (3k+1)(-4) - 8 = 0$

$\Longrightarrow$

$16k - 12k - 4 - 8 = 0$

$4k - 12 = 0$

$4k = 12$

$K = 3$

(b)

First we have the value of k, which is 4, So k=3

∴ By parts (a)

$\Longrightarrow$

$kx^2 + (3k+1)x + 8 = 0$

$3x^2 + 10x - 8 = 0$

Factorization

$3x^2 + 12x - 2x - 8 = 0$

$3x(x+4) - 2(x+4) = 0$

$(3x-2)(x+4) = 0$

$3x-2=0$ or $x+4 = 0$

$x = \frac{2}{3}$ or $x = -4$

So, the second possible value of x is:

$x = \frac{2}{3}$ and $x = -4$

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