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# PURE MATH

## ALGEBRA AND FUNCTION

| | |
|---|---|
| Level & Board | EDEXCEL (A-LEVEL) |
| TOPIC: | BINOMIAL EXPENSION |
| PAPER TYPE: | SOLUTION - 1 |
| TOTAL QUESTIONS | 8 |
| TOTAL MARKS | 39 |

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**Q.1**

(a) To demonstrate the given expression using the binomial expansion, we use the binomial theorem. The theorem states that for any real number n and any real number x, the expansion of (a + b)^n is given by:

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$$

In this case, we have a = 2, b = -x, and n = 1/2 (since we're dealing with the square root). Applying the binomial theorem, we get:

$$\sqrt{4-x} = (2-x)^{\frac{1}{2}}$$

Using the binomial expansion, we get:

$$\sqrt{4-x} = 2 - \frac{1}{2}x + \frac{1}{8}x^2 - - - -$$

(b) To check if the expansion is valid for x = 1, we must ensure that the absolute value of the ratio of consecutive terms approaches zero as we go further in the expansion.

Let's look at the ratio of the (k+1)-th term to the k-th term:

$$\left|\frac{a_{k+1}}{a_k}\right| = \left|\frac{\binom{\frac{1}{2}}{k+1} 2^{\frac{1}{2}-k-1}(-x)^{k+1}}{\binom{\frac{1}{2}}{k+1} 2^{\frac{1}{2}-k}(-x)^{k}}\right|$$

Simplifying this expression

$$\left|\frac{a_{k+1}}{a_k}\right| = \left|\frac{-x}{2(k+1)}\right|$$

The absolute value of this expression approaches zero as k becomes larger. Therefore, the expansion is valid for x = 1.

In conclusion, the binomial expansion is valid for x = 1, and substituting x = 1 into the given expression will provide an approximate value for $\sqrt{3}$.

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**Q.2**

$The\ binomial\ expansion\ for (1+2x)^{\frac{4}{3}}.Usig\ the\ binomial\ theorem\ is:$

$$(1+2x)^{\frac{4}{3}} = 1 + \frac{8}{3}x + \frac{16}{9}x^2 + ---$$

$Now,$

$Let's use\ this\ expansion\ to\ approximate$

$\sqrt[3]{10}\ by\ subsituting\ x = \frac{1}{2},$

$$1 + 2\left(\frac{1}{2}\right)^{\frac{1}{2}} = 1 + \frac{8}{3}\left(\frac{1}{2}\right) + \frac{16}{9}\left(\frac{1}{2}\right)^2 + ---$$

$Simplify$

$$(3)^{\frac{4}{3}} = 1 + \frac{4}{3} + \frac{4}{9}$$

$$\sqrt[3]{10} \approx 1 + \frac{4}{3} + \frac{4}{9}$$

$So, By\ using\ the\ binomial\ expansion, we\ have\ an\ approximation\ for$

$\sqrt[3]{10}\ as\ \frac{19}{9}.$

**Q.3**

$$(1.03)^{\frac{1}{3}} \quad = (1+003)^{\frac{1}{3}}$$

$$= 1 + \left(\frac{1}{3}\right)(0.03) + \frac{\left(\frac{1}{3}\right)\left(\frac{1}{3}-1\right)}{2!}(0.03)^2 + \cdots.$$

$$= 1 + 0.01 + \frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)}{2!}(0.03)^2 + \cdots$$

$$= 1 + 0.01 + \frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)}{2}(0.0009) = 1 + 0.01 - \frac{2}{18}(0.0009)$$

$$= 1 + 0.01 - \frac{1}{9}\big((0.0009)\big) = 1 + 0.01 - 0.0001 = 1.0099$$

$$= 1.010\ \ approx$$

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**Q.4**

$The\ binomial\ expansion\ for\ (1-4x)^{\frac{2}{3}}\ Using\ the\ binomial\ theorem\ is:$

$$(1-4x)^{\frac{2}{3}} = 1 - \frac{8}{3}x + \frac{16}{9}x^2 + ---$$

$Now,$

$Let's use\ this\ expansion\ to\ approximate \sqrt[3]{2}\ by\ subsituting\ x = \frac{1}{4}$

$$\left(1 - 4\left(\frac{1}{4}\right)\right)^{\frac{2}{3}} = 1 - \frac{8}{3}\left(\frac{1}{4}\right) + \left(\frac{16}{9}\right)\left(\frac{1}{4}\right)^2 + ---$$

$Simplifying:$

$$\left(\frac{1}{2}\right)^{\frac{2}{3}} = 1 - \frac{1}{3} + \frac{1}{36}$$

$$\sqrt[3]{2} \approx 1 - \frac{1}{3} + \frac{1}{36}$$

$So, by\ using\ the\ binomial\ expansion\ we\ have\ an\ approximation\ for$

$\sqrt[3]{2}\ as\ \frac{35}{36}.$

**Q.5**

$$\sqrt[4]{17} \qquad = (17)^{\frac{1}{4}}$$

$$= (16+1)^{\frac{1}{4}} = \left[16\left(1+\frac{1}{16}\right)\right]^{\frac{1}{4}}$$

$$= 16^{\frac{1}{4}}\left(1+\frac{1}{16}\right)^{\frac{1}{4}}$$

$$= (2^4)^{\frac{1}{4}}\left(1 + \left(\frac{1}{4}\right)\left(\frac{1}{16}\right) + \frac{\left(\frac{1}{4}\right)\left(\frac{1}{4}-1\right)}{2!}\left(\frac{1}{16}\right)^2 + \cdots\right)$$

$$= 2\left[1 + \frac{1}{64} + \frac{\left(\frac{1}{4}\right)\left(-\frac{3}{4}\right)}{2}\right] = 2\left[1 + \frac{1}{64} - \left(\frac{3}{32}\right)\left(\frac{1}{256}\right)\right]$$

$$= \left[1 + \frac{1}{64} - \frac{3}{8192}\right] = 2(1 + 0.0156 - 0.0004)$$

$$= 2(1.0152) = 2.0304$$

$$= 2.030 \quad approx$$


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**Q.6**

$The\ binomial\ expantion\ for\ (1-3x)^{\frac{4}{3}}\ Using\ the\ binomial\ theorem\ is:$

$$(1-3x)^{\frac{4}{3}} = 1 - 4\text{x} + \frac{18}{3}x^2 + ---$$

$Now,$

$Let's use\ this\ binomial\ expansion\ toapproximate\ \sqrt[3]{\frac{1}{2}}\ by\ subsituting\ x = \frac{1}{6}$

$$\left(1 - 3\left(\frac{1}{6}\right)\right)^{\frac{4}{3}} = 1 - 4\left(\frac{1}{6}\right) + \frac{18}{3}\left(\frac{1}{6}\right)^2 + ---$$

$Simplyfying:$

$$\left(\frac{1}{2}\right)^{\frac{1}{3}} = 1 - \frac{2}{3} + \frac{1}{18}$$

$$\sqrt[3]{\frac{1}{2}} \approx 1 - \frac{2}{3} + \frac{1}{18}$$

$So, by\ using\ the\ binomial\ expansion, we\ have\ an\ approximation\ for$

$\sqrt[3]{\frac{1}{2}}\ as\ \frac{17}{18}.$

**Q.7**

$The\ binomial\ expansion\ for\ (3+2x)^{\frac{5}{2}}\ Using\ the\ binomial\ theorem\ is:$

$$(3+2x)^{\frac{5}{2}} = 243 + 405x + 270x^2 + ---$$

$Now,$

$Let's use\ this\ expansion\ to\ approximate\ \sqrt{11}\ by\ substituting\ x = 4$

$$\left(3 + 2(4)\right)^{\frac{5}{2}} = 243 + 405(4) + 270(4)^2 + ---$$

$Simpolifying:$

$$(11)^{\frac{1}{2}} = 243 + 1620 + 4320$$

$$\sqrt{11} \approx 6183$$

$So, by\ using\ the\ binomial\ expansion, we\ have\ an\ opproximation\ for$

$\sqrt{11} = 6183.$


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**Q.8**

$The\ binormal\ expansion\ for\ (1-5x)^{\frac{2}{3}}\ Using\ the\ binormal\ theorem\ is:$

$$(1-5x)^{\frac{2}{3}} = 1 - \frac{10}{3}x + \frac{40}{9}x^2 + ---$$

$Now,$

$Let's use\ the\ expansion\ to\ approximate\ \sqrt[3]{\frac{1}{2}}\ by\ subsituting\ x = \frac{1}{5}$

$$\left(1 - 5\left(\frac{1}{5}\right)\right)^{\frac{2}{3}} = 1 - \frac{10}{3}\left(\frac{1}{5}\right) + \frac{40}{9}\left(\frac{1}{5}\right)^2 + ---$$

$Simplifying:$

$$\left(\frac{1}{2}\right)^{\frac{1}{3}} = 1 - \frac{2}{2} + \frac{4}{25}$$

$$\sqrt[3]{\frac{1}{2}} \approx 1 - \frac{2}{3} + \frac{4}{25}$$

$So, by\ using\ the\ binomial\ expansion, we\ have\ an\ approximation\ for$

$\sqrt[3]{\frac{1}{2}}\ as\ \frac{43}{45}.$


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