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PURE MATH

ALGEBRA AND FUNCTION

Level & Board	EDEXCEL (A-LEVEL)
TOPIC:	BINOMIAL EXPENSION
PAPER TYPE:	SOLUTION - 5
TOTAL QUESTIONS	8
TOTAL MARKS	39

Q.1

$$\sqrt[5]{31} = (31)^{\frac{1}{5}} = (33-1)^{\frac{1}{5}}$$

$$= \left[32\left(1-\frac{1}{32}\right)\right]^{\frac{1}{5}}$$

$$= 32^{\frac{1}{5}}\left(1-\frac{1}{32}\right)^{\frac{1}{5}}$$

$$= (2^5)^{\frac{1}{5}}\left(1+\left(\frac{1}{5}\right)\left(-\frac{1}{32}\right)+\frac{\left(\frac{1}{5}\right)\left(\frac{1}{5}-1\right)}{2!}-\left(\frac{1}{32}\right)^2+\cdots\right)$$

$$= 2\left[1-\frac{1}{160}+\frac{\left(\frac{1}{5}\right)\left(-\frac{4}{5}\right)}{2}\left(\frac{1}{1024}\right)\right] = 2\left[1-\frac{1}{160}-\left(\frac{4}{50}\right)\left(\frac{1}{1024}\right)\right]$$

$$= 2\left[1-\frac{1}{160}-\frac{1}{12800}\right] = (1-0.0063-0.0001)$$

$$= 2(0.9936) = 1.9872 = 1.987\ approx$$

Q.2

$$\frac{1}{\sqrt{998}} = (998)^{\frac{1}{3}} = (1000-2)^{\frac{1}{3}}$$

$$= \left[1000\left(1-\frac{2}{1000}\right)\right]^{\frac{1}{3}} = 1000^{\frac{1}{3}}\left(1-\frac{1}{500}\right)^{\frac{1}{3}}$$

$$= (10^3)^{\frac{1}{3}}\left(1+\left(-\frac{1}{3}\right)\left(-\frac{1}{500}\right)+\frac{\left(-\frac{1}{3}\right)\left(-\frac{1}{3}-1\right)}{2!}\left(\frac{1}{500}\right)^2+\cdots\right)$$

$$= \frac{1}{10}\left[1+\frac{1}{1500}+\frac{1}{1125000}\right] = \frac{1}{10}(1+0.0007+0)$$

$$= \frac{1}{10}(1{,}001) = 0.1001$$

$$= 0.100 \quad approx$$

Q.3

$$\frac{1}{\sqrt{252}} = (252)^{\frac{1}{5}} = (243+9)^{\frac{1}{5}}$$

$$= \left[243\left(1+\frac{9}{243}\right)\right]^{\frac{1}{5}} = 243^{\frac{1}{5}}\left(1+\frac{1}{27}\right)^{\frac{1}{5}}$$

$$= (3^5)^{\frac{1}{5}}\left(1+\left(-\frac{1}{5}\right)\left(\frac{1}{27}\right)+\frac{\left(-\frac{1}{5}\right)\left(-\frac{1}{5}-1\right)}{2!}\left(\frac{1}{27}\right)^2+\cdots\right)$$

$$= 3^{-1}\left[1-\frac{1}{135}+\frac{\left(-\frac{1}{5}\right)\left(-\frac{6}{5}\right)}{2}\left(\frac{1}{729}\right)\right] = \frac{1}{3}\left[1-\frac{1}{135}+\left(\frac{6}{50}\right)\left(\frac{1}{729}\right)\right]$$

$$= \frac{1}{3}\left[1-\frac{1}{135}+\frac{1}{6074}\right] = \frac{1}{3}(1-0.0074+0.0002)$$

$$= \frac{1}{3}(0.9928) = 0.3309$$

$$= 0.331 \quad approx$$

Q.4

$$\frac{\sqrt{7}}{\sqrt{8}} = \sqrt{\frac{7}{8}} = \sqrt{1-\frac{1}{8}} = \left(1-\frac{1}{8}\right)^{\frac{1}{2}}$$

$$= 1+\left(\frac{1}{2}\right)\left(-\frac{1}{8}\right)+\frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)}{2!}\left(-\frac{1}{8}\right)^2+\cdots$$

$$= 1-\frac{1}{16}+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2}\left(\frac{1}{64}\right)+\cdots$$

$$= 1-\frac{1}{16}-\left(\frac{1}{8}\right)\left(\frac{1}{64}\right) = 1-\frac{1}{16}-\frac{1}{512}$$

$$= 1-0.0625-0.0020$$

$$= 0.9355$$

$$= 0.936 \quad approx$$

Q.5

$$(0.998)^{-\frac{1}{3}} = (1-0.002)^{-\frac{1}{3}}$$

$$= \left(1 + \left(-\frac{1}{3}\right)(-0.002) + \frac{\left(-\frac{1}{3}\right)\left(-\frac{1}{3}-1\right)}{2!}(-0.002)^{2+\cdots}\right)$$

$$= \left[1 + 0.001 + \frac{\left(-\frac{1}{3}\right)\left(-\frac{4}{3}\right)}{2}(0.000004)\right] = \left[1 + 0.0000666 + \frac{2}{9}(0.000004)\right]$$

$$= 1.0007$$

$$= 1.001 \ \ approx$$

Q.6

$$\frac{1}{\sqrt[6]{486}} = (486)^{-\frac{1}{6}}$$

$$= (729-243)^{\frac{1}{6}} = \left[729\left(1-\frac{243}{729}\right)\right]^{\frac{1}{6}}$$

$$= 729^{-\frac{1}{6}}\left(1-\frac{1}{3}\right)^{-\frac{1}{6}}$$

$$= (3^6)^{-\frac{1}{6}}\left(1 + \left(-\frac{1}{6}\right)\left(-\frac{1}{3}\right) + \frac{\left(-\frac{1}{6}\right)\left(-\frac{1}{6}-1\right)}{2!}\left(-\frac{1}{3}\right)^2 + \cdots\right)$$

$$= 3^{-1}\left[1 + \frac{1}{18} + \frac{\left(-\frac{1}{6}\right)\left(-\frac{17}{6}\right)}{2}\left(\frac{1}{9}\right)\right] = 3^{-1}\left[1 + \frac{1}{18} + \left(\frac{7}{72}\right)\left(\frac{1}{9}\right)\right]$$

$$= \frac{1}{3}\left[1 + \frac{1}{18} + \frac{7}{648}\right] = \frac{1}{3}(1 + 0.0556 + 0.0108)$$

$$= \frac{1}{3}(1.0664) = 0.3555$$

$$= 0.3556 \ \ approx$$

Q.7

$$(280)^{\frac{1}{4}} = (1296 - 16)^{\frac{1}{4}}$$

$$= \left(1296^{\frac{1}{4}}\left(1 - \frac{16}{1296}\right)\right)^{\frac{1}{4}} = 1296^{\frac{1}{4}}\left(1 - \frac{1}{81}\right)^{\frac{1}{4}}$$

$$= (6^4)^{\frac{1}{4}}\left(1 + \left(\frac{1}{4}\right)\left(-\frac{1}{81}\right) + \frac{\left(\frac{1}{4}\right)\left(\frac{1}{4}-1\right)}{2\ \ !}\left(-\frac{1}{81}\right)^2 + \cdots\right)$$

$$= 6\left[1 - \frac{1}{324} + \frac{\left(\frac{1}{4}\right)\left(-\frac{3}{4}\right)}{2}\frac{1}{6561}\right] = 6\left[1 - \frac{1}{324} - \left(\frac{3}{32}\right)\left(\frac{1}{6561}\right)\right]$$

$$= 6\left[1 - \frac{1}{324} - \frac{1}{69384}\right] = 6(1 - 0.003086 - 0.000014)$$

$$= 6(0.9969) = 5.9814 \quad approx$$

Q.8

$$(1-x)^{\frac{1}{2}} = 1 + \left(\frac{1}{2}\right)(-x) + \frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)}{2\ \ !}(-x)^2 + \frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)}{3\ \ !}(-x)^3 + \cdots$$

$$= 1 + \left(\frac{1}{2}\right)(-x) + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2}x^2\,\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{6}x^3 + \cdots$$

$$= 1 + \left(\frac{1}{2}\right)(-x) + \frac{1}{4}.\frac{1}{2}x^2 - \frac{3}{8}.\frac{1}{6}x^3 + \cdots$$

$$(1-x)^{\frac{1}{2}} = 1 - \frac{1}{2}x - \frac{1}{8}x^2 - \frac{1}{16}x^3 + \cdots \; is\ valid\ if\ |x| < 1$$

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