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PURE MATH

ALGEBRA AND FUNCTION

Level & Board	EDEXCEL (A-LEVEL)
TOPIC:	BINOMIAL EXPENSION
PAPER TYPE:	SOLUTION - 4
TOTAL QUESTIONS	8
TOTAL MARKS	39

1.

$$\sqrt{99} = (99)^{\frac{1}{2}}$$

$$= (100-1)^{\frac{1}{2}} = \left[100\left(1-\frac{1}{100}\right)\right]^{\frac{1}{2}} = 100^{\frac{1}{2}}\left(1-\frac{1}{100}\right)^{\frac{1}{2}}$$

$$= 10100(1-0.01)^{\frac{1}{2}}$$

$$= 10\left(1+\left(\frac{1}{2}\right)(-0.01)+\frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)-1}{2!}(-0.01)^2+\cdots.\right)$$

$$= 10\left[1-0.005+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2}(0.0001)\right] = 10\left[1-0.005-\frac{1}{8}(0.0001)\right]$$

$$= 10(1-0.0050) = 10(1.995) = 9{,}950\ approx.$$

2.

$$(0.98)^{\frac{1}{2}} \qquad = (1-0.02)^{\frac{1}{2}}$$

$$= 1+\left(\frac{1}{2}\right)(-0.02)+\frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)}{2!}(-0.02)^2+\cdots$$

$$= 1-0.01+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2}(0.0004) = 1-0.01-\frac{1}{8}(0.0004)$$

$$= 1-0.01-0.0001$$

$$= 0.990\ approx$$

3.

$$(1.03)^{\frac{1}{3}} \qquad = (1+003)^{\frac{1}{3}}$$

$$= 1+\left(\frac{1}{3}\right)(0.03)+\frac{\left(\frac{1}{3}\right)\left(\frac{1}{3}-1\right)}{2!}(0.03)^2+\cdots.$$

$$= 1+0.01+\frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)}{2!}(0.03)^2+\cdots$$

$$= 1 + 0.01 + \frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)}{2}(0.0009) = 1 + 0.01 - \frac{2}{18}(0.0009)$$

$$= 1 + 0.01 - \frac{1}{9}\big((0.0009)\big) = 1 + 0.01 - 0.0001 = 1.0099$$

$$= 1.010 \;\; approx$$

4.

$$\sqrt[3]{65} \;\; = (65)^{\frac{1}{3}}$$

$$= (64 + 1)^{\frac{1}{3}} = \left[64\left(1 + \frac{1}{64}\right)\right]^{\frac{1}{3}} = 64^{\frac{1}{3}}\left[1 + \frac{1}{64}\right]^{\frac{1}{3}}$$

$$= (4^3)^{\frac{1}{3}}\left[1 + \left(\frac{1}{3}\right)\left(1 + \frac{1}{64}\right) + \frac{\left(\frac{1}{3}\right)\left(\frac{1}{3}-1\right)}{2!}\left(\frac{1}{64}\right)^2 + \cdots\right]$$

$$= 4\left[1 + \frac{1}{192+=} + \frac{\left(\frac{1}{3}\right)\left(\frac{-2}{3}\right)}{2}\left(\frac{1}{4096}\right)\right] = 4\left[1 + \frac{1}{192} - \left(\frac{2}{18}\right)\left(\frac{1}{4096}\right)\right]$$

$$= 4\left[1 + \frac{1}{192} - \frac{1}{36864}\right] = 4(1 + 0.0052 - 0)$$

$$= 4(1.005) = 4.0208$$

$$= 4.021 \;\; approx$$

5.

$$\sqrt[4]{17} = (17)^{\frac{1}{4}}$$

$$= (16 + 1)^{\frac{1}{4}} = \left[16\left(1 + \frac{1}{16}\right)\right]^{\frac{1}{4}}$$

$$= 16^{\frac{1}{4}}\left(1 + \frac{1}{16}\right)^{\frac{1}{4}}$$

$$= (2^4)^{\frac{1}{4}}\left(1 + \left(\frac{1}{4}\right)\left(\frac{1}{16}\right) + \frac{\left(\frac{1}{4}\right)\left(\frac{1}{4}-1\right)}{2!}\left(\frac{1}{16}\right)^2 + \cdots\right)$$

$$= 2\left[1 + \frac{1}{64} + \frac{\left(\frac{1}{4}\right)\left(-\frac{3}{4}\right)}{2}\right] = 2\left[1 + \frac{1}{64} - \left(\frac{3}{32}\right)\left(\frac{1}{256}\right)\right]$$

$$= \left[1 + \frac{1}{64} - \frac{3}{8192}\right] = 2(1 + 0.0156 - 0.0004)$$

$$= 2(1.0152) = 2.0304$$

$$= 2.030 \quad approx$$

Q.6

$The\ binomial\ expantion\ for\ (1-3x)^{\frac{4}{3}}\ Using\ the\ binomial\ theorem\ is:$

$$(1-3x)^{\frac{4}{3}} = 1 - 4\text{x} + \frac{18}{3}x^2 + \ ---$$

$Now,$

$Let's use\ this\ binomial\ expansion\ toapproximate\ \sqrt[3]{\frac{1}{2}}\ by\ subsituting\ x = \frac{1}{6}$

$$\left(1 - 3\left(\frac{1}{6}\right)\right)^{\frac{4}{3}} = 1 - 4\left(\frac{1}{6}\right) + \frac{18}{3}\left(\frac{1}{6}\right)^2 + \ ---$$

$Simplyfying:$

$$\left(\frac{1}{2}\right)^{\frac{1}{3}} = 1 - \frac{2}{3} + \frac{1}{18}$$

$$\sqrt[3]{\frac{1}{2}} \approx 1 - \frac{2}{3} + \frac{1}{18}$$

$So, by\ using\ the\ binomial\ expansion, we\ have\ an\ approximation\ for$

$\sqrt[3]{\frac{1}{2}}\ \ as\ \ \frac{17}{18}.$

Q.7

$The\ binomial\ expansion\ for\ (3+2x)^{\frac{5}{2}}\ Using\ the\ binomial\ theorem\ is:$

$$(3+2x)^{\frac{5}{2}} = 243 + 405x + 270x^2 + ---$$

$Now,$

$Let's use\ this\ expansion\ to\ approximate\ \sqrt{11}\ by\ substituting\ x = 4$

$$\left(3+2(4)\right)^{\frac{5}{2}} = 243 + 405(4) + 270(4)^2 + ---$$

$Simpolifying:$

$$(11)^{\frac{1}{2}} = 243 + 1620 + 4320$$

$$\sqrt{11} \approx 6183$$

$So, by\ using\ the\ binomial\ expansion, we\ have\ an\ opproximation\ for$

$\sqrt{11} = 6183.$

Q.8

$The\ binormal\ expansion\ for\ (1-5x)^{\frac{2}{3}}\ Using\ the\ binormal\ theorem\ is:$

$$(1-5x)^{\frac{2}{3}} = 1 - \frac{10}{3}x + \frac{40}{9}x^2 + ---$$

$Now,$

$Let's use\ the\ expansion\ to\ approximate\ \sqrt[3]{\frac{1}{2}}\ by\ subsituting\ x = \frac{1}{5}$

$$\left(1 - 5\left(\frac{1}{5}\right)\right)^{\frac{2}{3}} = 1 - \frac{10}{3}\left(\frac{1}{5}\right) + \frac{40}{9}\left(\frac{1}{5}\right)^2 + ---$$

$Simplifying:$

$$\left(\frac{1}{2}\right)^{\frac{1}{3}} = 1 - \frac{2}{2} + \frac{4}{25}$$

$$\sqrt[3]{\frac{1}{2}} \approx 1 - \frac{2}{3} + \frac{4}{25}$$

$So, by\ using\ the\ binomial\ expansion, we\ have\ an\ approximation\ for$

$\sqrt[3]{\frac{1}{2}}\ as\ \frac{43}{45}.$

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