5.1 Hypothesis Testing

Question Paper

Course	OCR A Level Maths: Statistics
Section	5. Hypothesis Testing
Topic	5.1 Hypothesis Testing
Difficulty	Medium

Time allowed: **50**

Score: **/41**

Percentage: **/100**

Question 1

(a) Explain what you understand by a critical region of a test statistic.

[2 marks]

Question 1

Nationally 44% of A Level mathematics students identify as female. The headteacher of a particular school claims that the proportion of A Level mathematics students in the school who identify as female is higher than the national average.

(b) (i) State a suitable null hypothesis to test the headteacher's claim.

(ii) State a suitable alternative hypothesis to test the headteacher's claim.

[2 marks]

Question 1

The headteacher takes a random sample of 60 A Level mathematics students and records the number of them who identify as female, x. For a test at the 10% significance level the critical region is $X \geq 32$.

(c) Given that $x = 36$, comment on the headteacher's claim.

[2 marks]

Question 2

The probability of a chicken laying an egg on any given day is 65%. Two farmers, Amina and Bert, have 30 chickens each. They believe that the probability of their chickens laying an egg on any given day is different to 65%.

(a) (i) State a suitable null hypothesis to test the farmers' belief.

(ii) State a suitable alternative hypothesis for a two-tailed test.

[2 marks]

Question 2

During a specific day, Amina and Bert each record the number of their 30 chickens that lay an egg. At the 5% significance level the critical regions for this test are $X \leq 13$ and $X \geq 25$.

(b) Write down the critical values for the hypothesis test.

[1 mark]

Question 2

(c) (i) Given that for Amina $x = 12$, comment on her belief.

(ii) Given that for Bert $x = 24$, comment on his belief.

[4 marks]

Question 3

A memory experiment involves having participants read a list of 20 words for two minutes and then recording how many of the words they can recall. Peter, a psychologist, claims that more than 60% of teenagers can recall all the words. Peter takes a random sample of 40 teenagers and records how many of them recall all the words.

(a) (i) State a suitable null hypothesis to test the psychologist's claim.

(ii) State a suitable alternative hypothesis to test the psychologist's claim.

[2 marks]

Question 3

(b) Given that the critical value for the test is $x = 19$, state the outcome of the test if

(i) 18 out of the 40 teenagers recall all the words

(ii) 19 out of the 40 teenagers recall all the words

(iii) 20 out of the 40 teenagers recall all the words.

[3 marks]

Question 4

A machine produces toys for a company. It was found that 8% of the toys it was producing were faulty. After an engineer works on the machine, she claims that the proportion of faulty toys should now have decreased.

(a) State suitable null and alternative hypotheses to test this claim.

[2 marks]

Question 4

After the engineer is finished, the manager of the company takes a random sample of 100 toys and finds that 2 of them are faulty.

(b) Given that $\mathrm{P}(X \leq 2) = 0.01127$ when $X \sim \mathrm{B}(100, 0.08)$, determine the outcome of the hypothesis test using a 1% level of significance. Give your conclusion in context.

[2 marks]

Question 5

After it was estimated that only 72% of patients were turning up for their appointments at Pearly Teeth dental surgery, the owner began sending text message reminders to the patients on the day before their appointments. In order to test whether the reminders have increased the proportion of patients turning up to their appointments, the owner decides to conduct a hypothesis test at the 5% level of significance using the next 160 patients scheduled for appointments as a sample.

(a) State suitable null and alternative hypotheses to test this claim.

[2 marks]

Question 5

(b) Describe, in context, the test statistic for this test.

[1 mark]

Question 5

Out of the 160 patients used for the sample, 127 turned up for their appointments. The p-value for $x = 127$ is 0.02094.

(c) Determine the outcome of the hypothesis test, giving your conclusion in context.

[2 marks]

Question 6

Chase buys a board game which contains a six-sided dice. He rolls the dice 150 times and obtains the number six on 15 occasions. Chase wishes to test his belief that the dice is not fair.

(a) (i) State a suitable null hypothesis to test Chase's belief.

(ii) State a suitable alternative hypothesis for a two-tailed test.

[2 marks]

Question 6

(b) Given that $P(X \leq 15) = 0.01452$ when $X \sim B\left(150, \frac{1}{6}\right)$, test Chase's belief that the dice is not fair, using a 2% level of significance.

[3 marks]

Question 7

A test of the null hypothesis $H_0: p = 0.3$ is carried out for the random variable $X \sim \mathrm{B}(25, p)$. The observed value of the test statistic is $x = 3$. You are given the following probabilities:

$$\mathrm{P}(X < 3) = 0.00896$$
$$\mathrm{P}(X = 3) = 0.02428$$
$$\mathrm{P}(X \leq 3) = 0.03324$$

Determine the outcome of the test, with reasons, when the alternative hypothesis is:

(a) $H_1: p < 0.3$ with a 1% level of significance.

[2 marks]

Question 7

(b) $H_1: p \neq 0.3$ with a 5% level of significance.

[2 marks]

Question 8

A two-tailed test of the null hypothesis $H_0: p = 0.23$ is carried out for the random variable $X \sim \mathrm{B}(60, p)$.

(a) Write down the alternative hypothesis.

[1 mark]

Question 8

One of the critical regions is $X \geq 20$. You are given the following probabilities:

$$P(X \leq 8) = 0.04603$$
$$P(X \leq 9) = 0.08932$$
$$P(X \leq 10) = 0.15526$$

(b) Given that a 10% level of significance is used, determine the other critical region. Give a reason for your answer by using a relevant probability.

[2 marks]

Question 8

You are also given that $P(X \geq 20) = 0.04427$.

(c) Find the actual level of significance of this test.

[2 marks]