5.2 Hypothesis Testing (Binomial Distribution)

Question Paper

Course	OCR A Level Maths: Statistics
Section	5. Hypothesis Testing
Topic	5.2 Hypothesis Testing (Binomial Distribution)
Difficulty	Medium

Time allowed: 60

Score: /48

Percentage: /100

Question 1

A single observation is taken from a discrete random variable $X \sim B(35, 0.4)$ to test $H_0: p = 0.4$ against $H_1: p < 0.4$.

(a) Using a 5% level of significance, find the critical region for this test.

[3 marks]

Question 1

The actual value for the observation was 10.

(b) State a conclusion to the hypothesis test for this value, giving a reason for your answer.

[2 marks]

Question 2

Harry is using the random variable $X \sim B(40, 0.55)$ to test the hypotheses:

$$H_0: p = 0.55$$
$$H_1: p \neq 0.55$$

Harry states that the critical regions are $X \leq 17$ and $X \geq 27$.

(a) (i) Calculate the probability of incorrectly rejecting the null hypothesis.

(ii) State, with a reason, the conclusion of Harry's test given that a value of $x = 18$ is observed for the test statistic.

[4 marks]

Question 2

Sally is using the random variable $Y \sim B(75, 0.3)$ to test the hypotheses:

$$H_0: p = 0.3$$
$$H_1: p > 0.3$$

Sally observes the value $y = 30$ for her test statistic.

(b) (i) Calculate the p-value of the observed test statistic $y = 30$.

(ii) State, with a reason, the conclusion of Sally's test if a 5% level of significance is used.

[4 marks]

Question 3

Charlie, the owner of a chocolate shop, claims that more than 60% of people can tell the difference between two brands of chocolate. Charlie takes a random sample of 150 customers and asks them to taste both brands of chocolate. He records that 103 of them could successfully tell the difference between the two brands of chocolate.

(a) State suitable null and alternative hypotheses to test Charlie's claim.

[2 marks]

Question 3

(b) Test, at the 10% level of significance, whether Charlie's claim is justified.

[3 marks]

Question 4

Nationally it is reported that four out of five people are right-handed. Edward, an education researcher, takes a random sample of 30 children under the age of 18 years old and records the number of them, X, who write with their right hand.

(a) If the national proportion applies to the sample, write down a suitable distribution for X.

[1 mark]

Question 4

Edward believes that the proportion of right-handed children differs from the national proportion for all people. To test his belief, he uses his sample of 30 children.

(b) State suitable null and alternative hypotheses to test Edward's belief.

[2 marks]

Question 4

(c) Using a 10% level of significance, find the critical values for a two-tailed test for Edwards' belief. You should state the probability of rejection for each tail, which should be less than 0.05 for each.

[3 marks]

Question 4

(d) Find the actual level of significance of a test based on your critical values.

[1 mark]

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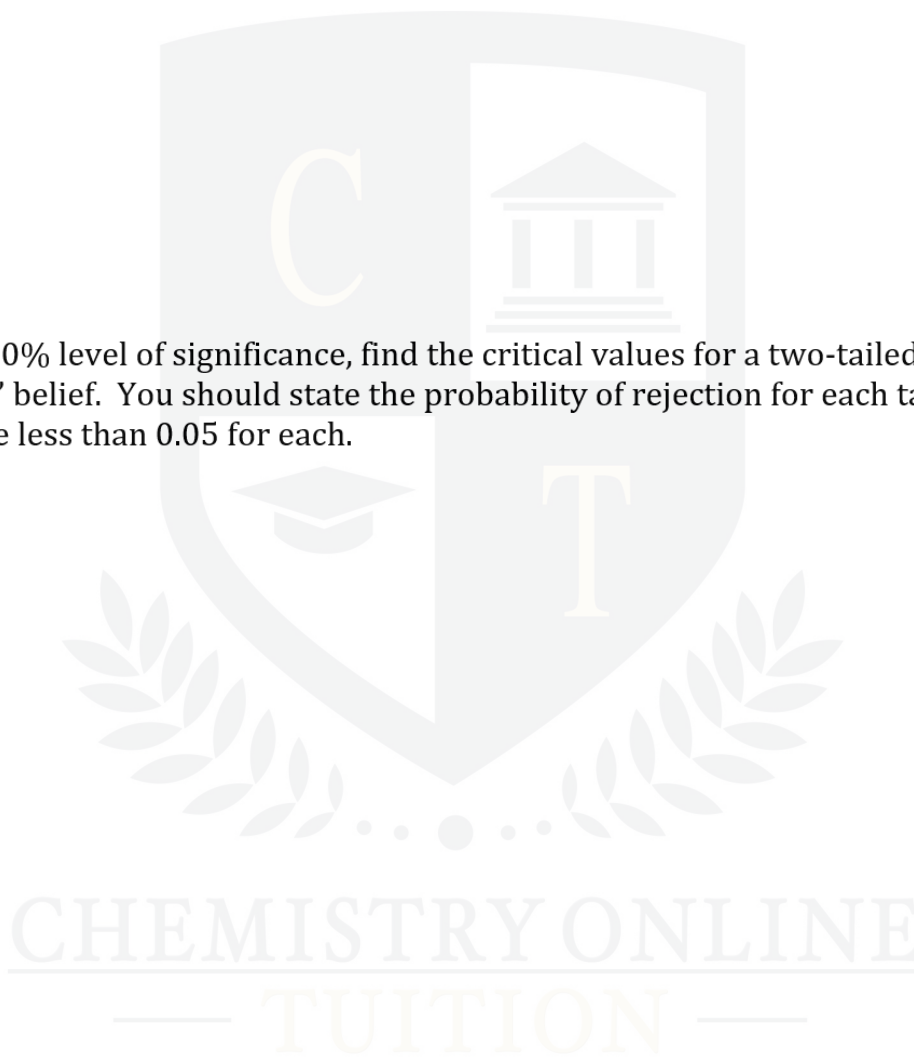
Question 4

Out of the 30 children in the sample, Edward recorded that 20 of them write with their right hand.

(e) Comment on Edward's belief based on this observation.

[1 mark]

Question 5

The existing treatment for a disease is known to be effective in 73% of cases. Dr Sabir develops a new treatment which she claims is more effective than the existing one. To test her claim she uses the new treatment on a sample of 60 patients with the disease and uses a binomial distribution to model the number of them who are cured.

(a) Explain two assumptions that Dr Sabir has made when using a binomial distribution to model the number of patients cured by the vaccine.

[2 marks]

Question 5

Dr Sabir notes that her treatment was effective for 51 out of the 60 patients used in the sample.

(b) Test, at the 1% level of significance, the validity of Dr Sabir's claim that her treatment is more effective than the existing one. State your hypotheses clearly.

[5 marks]

Question 5

(c) State the conclusion you would have reached if a 5% level of significance had been used for this test.

[1 mark]

Question 6

A "double yolker" is an egg which contains two yolks. It is known that the probability of a chicken laying a double yolker is 0.1%. A chicken farmer, Paolo, claims that double yolkers are rarer than the stated 0.1%. To test his claim, Paolo records that his chickens lay 1217 eggs in a month and he uses these as his sample. He discovers that none of these eggs are double yolkers.

(a) Test, at the 5% level of significance, whether there is evidence to support Paolo's claim that double yolkers are rarer than 0.1%. State your hypotheses clearly.

[4 marks]

Question 6

Paolo decides to take a larger sample so extends his test to three months. During this time, a sample of 3425 eggs is formed and none of them are double yolkers.

(b) Show that there is evidence, at the 5% level of significance, to support Paolo's claim that double yolkers are rarer than 0.1%.

[2 marks]

Question 6

(c) Paolo concludes that the probability of a double yolker is **definitely** less than 0.1%. Give a reason to explain whether Paolo's conclusion is justified.

[1 mark]

Question 7

It is known that 61% of male dragons eat more than 20 sheep within a day. Bill, a dragon breeder, suspects that the proportion of female dragons that eat more than 20 sheep within a day is different to males.

(a) State suitable null and alternative hypotheses for a two-tailed test for Bill's suspicion.

[2 marks]

Question 7

To test his suspicion, Bill observes a random sample of 80 female dragons during a full day and counts how many sheep they eat. He finds that 40 out of the 80 female dragons ate more than 20 sheep within the day.

(b) Test, at the 5% level, whether there is evidence to support Bill's suspicion.

[3 marks]

Question 7

(c) Determine the outcome of the test, at the 5% level, if Bill had used a one-tailed test to check whether the proportion of females eating more than 20 sheep within a day is lower than the proportion of males.

[2 marks]