

## 5.3 Hypothesis Testing (Normal Distribution) (A Level only)

### Question Paper

Course	OCR A Level Maths: Statistics
Section	5. Hypothesis Testing
Topic	5.3 Hypothesis Testing (Normal Distribution) (A Level only)
Difficulty	Medium

Time allowed:

60

Score:

/46

Percentage:

/100

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### Question 1

The gestation period of a female kangaroo,  $X$ , can be modelled as a normal distribution with a mean of 29 days and a standard deviation of 4 days.

- (a) Given that a randomly selected female kangaroo is pregnant, find the probability that the gestation period will be between 25 and 32 days.

[1 mark]

### Question 1

A random sample of 16 pregnant kangaroos is taken and the mean of their gestation periods is calculated.

- (b) (i) Write down the distribution of the sample mean,  $\bar{X}$ ,  
(ii) Calculate the probability that the sample mean of the 16 gestation periods is between 25 and 32 days.

[2 marks]

### Question 2

For the video game *Super Maria*, it is known that the length of time,  $T$  minutes, it takes a gamer to complete the final level of the game can be modelled as a normal distribution with  $T \sim N(57.2, 5^2)$ .

- (a) Find the interquartile range for the times taken to complete the final level of *Super Maria*.

[2 marks]

## Question 2

During a *Super Maria* competition, gamers are randomly put into teams of 9 and each member plays the final level. The mean time for each team is calculated and prizes are given to teams whose means are in the fastest 10% of mean times.

- (b) (i) Write down the distribution of the sample mean,  $\bar{T}$ .
- (ii) Find, to the nearest second, the maximum mean time that would lead to a team winning a prize. Give your answer in minutes and seconds.

[4 marks]

## Question 3

The mass of a Burmese cat,  $C$ , follows a normal distribution with a mean of 4.2 kg and a standard deviation 1.3 kg. Kamala, a cat breeder, claims that Burmese cats weigh more than the average if they live in a household which contains young children. To test her claim, Kamala takes a random sample of 25 cats that live in households containing young children.

The null hypothesis,  $H_0: \mu = 4.2$ , is used to test Kamala's claim.

- (a) (i) Write down the alternative hypothesis to test Kamala's claim.
- (ii) Write down the distribution of the sample mean,  $\bar{C}$ .

[2 marks]

### Question 3

(b) Using a 5% level of significance, find the critical region for this test.

**[2 marks]**

### Question 3

Kamala calculates the mean of the 25 cats included in her sample to be 4.65 kg.

(c) Determine the outcome of the hypothesis test at the 5% level of significance, giving your answer in context.

**[2 marks]**

### Question 4

The time,  $X$  seconds, that it takes Pierre to run a 400 m race can be modelled using  $X \sim N(87, 16)$ . Pierre changes his diet and claims that the time it takes him to run 400 m has decreased.

(a) Write suitable null and alternative hypotheses to test Pierre's claim.

**[1 mark]**

#### Question 4

After changing his diet, Pierre runs 36 separate 400 m races and calculates his mean time on these races to be 86.1 seconds.

- (b) Use these 36 races as a sample to test, at the 5% level of significance, whether there is evidence to support Pierre's claim.

[4 marks]

#### Question 4

- (c) Give a reason to explain why the 36 races might not form a suitable sample for this test.

[1 mark]

#### Question 5

The average length,  $L$ , of a unicorn's horn is 91 cm with a variance of  $5 \text{ cm}^2$ . Luna researches unicorns and believes that unicorns that were born beneath a rainbow have longer horns. To test her belief, Luna takes a random sample of 12 unicorns that were born beneath a rainbow and measures the length of their horns.

- (a) (i) Write suitable null and alternative hypotheses to test Luna's claim.
- (ii) What assumption do we need to make about the length of unicorn horns so that a normal distribution can be used for the mean of the sample,  $\bar{L}$ .

[2 marks]

### Question 5

(b) Given that the critical value for the hypothesis test is 92.1 cm, calculate the level of significance for the test.

[3 marks]

### Question 6

The IQ of a student at Calculus High can be modelled as a random variable with the distribution  $N(126, 50)$ . The headteacher decides to play classical music during lunchtimes and suspects that this has caused a change in the average IQ of the students.

(a) Write suitable null and alternative hypotheses to test the headteacher's suspicion.

[1 mark]

### Question 6

The headteacher selects 10 students and asks them to complete an IQ test. Their scores are:

127, 127, 129, 130, 130, 132, 132, 132, 133, 138

(b) Test, at the 5% level of significance, whether there is evidence to support the headteacher's suspicion.

[5 marks]

### Question 6

It was later discovered that the 10 students used in the sample were all in the same advanced classes.

(c) Comment on the validity of the conclusion of the test based on this information.

[1 mark]

### Question 7

Carol is a new employee at a company and wishes to investigate whether there is a difference in pay based on gender, but she does not have access to information for all the employees. It is known that the average salary of a male employee is £32500, and it can be assumed the salary of a female employee follows a normal distribution with a standard deviation of £6100. Carol forms a sample using 20 randomly selected female employees.

(a) Write suitable null and alternative hypotheses to test whether the average salary of a female employee is different to the average salary of a male employee.

[1 mark]

### Question 7

(b) Using a 5% level of significance, find the critical regions for the test.

[3 marks]

### Question 7

The total of the salaries of the 20 employees used in the sample is £602 000.

- (c) Use this information to state a conclusion for Carol's investigation into pay differences based on gender.

[2 marks]

### Question 7

- (d) Would the outcome of the test have been different if a 10% level of significance had been used?

[2 marks]

### Question 8

- (a) Given that  $Z \sim N(0, 1^2)$ , find the value of  $d$  such that  $P(Z > d) = 0.1$ , correct to 4 decimal places.

[1 mark]



### Question 8

The population mean of the random variable  $X \sim N(\mu, 5^2)$  is being tested using a null hypothesis  $H_0: \mu = 20$  against the alternative hypothesis  $H_1: \mu > 20$ . A random sample of  $n$  observations is taken from the population and the sample mean is calculated as 22.

(b) Using a 10% level of significance, the null hypothesis is rejected. Find the smallest possible value of the sample size  $n$ .

**[4 marks]**

