4.2 Binomial Distribution

Question Paper

Course	OCR A Level Maths: Statistics
Section	4. Statistical Distributions
Topic	4.2 Binomial Distribution
Difficulty	Medium

Time allowed: 50

Score: /40

Percentage: /100

Question 1

A fair coin is tossed 20 times and the number of times it lands heads up is recorded.

(a) Define a suitable distribution to model the number of times the coin lands heads up, and justify your choice.

[2 marks]

Question 1

(b) Find the probability that the coin lands heads up 15 times.

[2 marks]

Question 2

For a jellyfish population in a certain area of the ocean, there is a 95% chance that any given jellyfish contains microplastic particles in its body.

(a) State any assumptions that are required to model the number of jellyfish containing microplastic particles in their bodies in a sample of size n as a binomial distribution.

[2 marks]

Question 2

(b) Using this model, for a sample size of 40, find the probability of

    (i)   exactly 38 jellyfish

    (ii)   all the jellyfish

having microplastic particles in their bodies.

[3 marks]

Question 3

Giovanni is rolling a biased dice, for which the probability of landing on a two is 0.25. He rolls the dice 10 times and records the number of times that it lands on a two. Find the probability that

    (i)   the dice lands on a two 4 times

    (ii)   the dice lands on a two 4 times, with the fourth two occurring on the final roll.

[4 marks]

Question 4

For cans of a particular brand of soft drink labelled as containing 330 ml, the actual volume of soft drink in a can varies. Although the company's quality control assures that the mean volume of soft drink in the cans remains at 330 ml, it is known from experience that the probability of any particular can of the soft drink containing less than 320 ml is 0.0296.

Tilly buys a pack of 24 cans of this soft drink. It may be assumed that those 24 cans represent a random sample. Let L represent the number of cans in the pack that contain less than 320 ml of soft drink.

(a) Write down the probability distribution that describes L.

[2 marks]

Question 4

(b) Find the probability that

    (i)   none of the cans

    (ii)   exactly two of the cans

    (iii) at least two of the cans

contain less than 320 ml of soft drink.

[4 marks]

Question 5

The random variable $X \sim B(40, 0.15)$. Find:

    (i) $P(X < 10)$

    (ii) $P(X > 7)$

    (iii) $P(3 \leq X < 14)$

    (iv) $P(5 < X < 12)$.

[4 marks]

Question 6

The random variable $X \sim B(40, 0.25)$. Find:

    (i)   the largest value of k such that $P(X < k) < 0.10$

    (ii)   the smallest value of r such that $P(X \geq r) < 0.05$

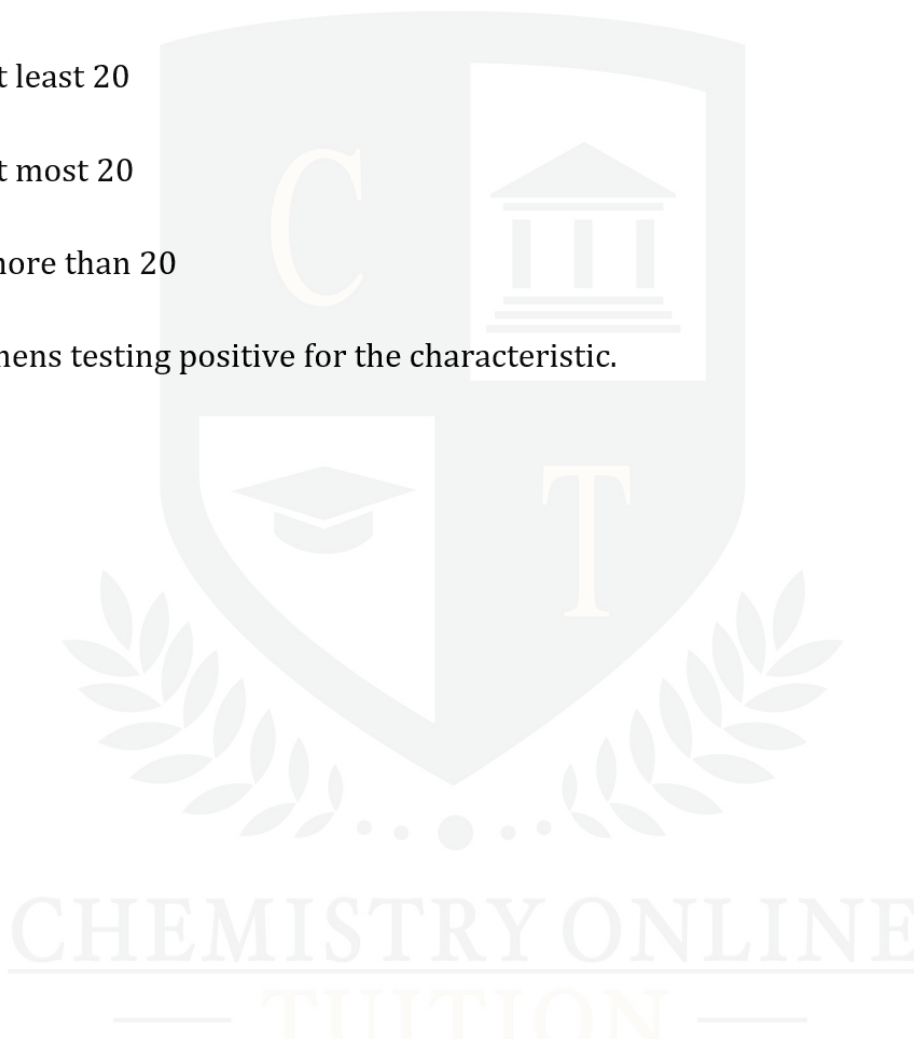   (iii) the largest value of s such that $P(X > s) > 0.95$.

[5 marks]

Question 7

In an experiment, the number of specimens testing positive for a certain characteristic is modelled by the random variable $X \sim B(50, 0.35)$. Find the probability of

(i)   fewer than 20

(ii)   no more than 20

(iii)   at least 20

(iv)   at most 20

(v)   more than 20

of the specimens testing positive for the characteristic.

[5 marks]

Question 8

In the town of Wooster, Ohio, it is known that 90% of the residents prefer the locally produced Woostershire brand sauce when preparing a Caesar salad. The other 10% of residents prefer another well-known brand.

30 residents are chosen at random by a pollster. Let the random variable X represent the number of those 30 residents that prefer Woostershire brand sauce.

(a) Suggest a suitable distribution for X and comment on any necessary assumptions.

[2 marks]

Question 8

(b) Find the probability that

   (i)   90% or more of the residents chosen prefer Woostershire brand sauce

   (ii)   none of the residents chosen prefer the other well-known brand.

[3 marks]

Question 8

The pollster knows that there is a greater than 97% chance of at least k of the 30 residents preferring Woostershire brand sauce, where k is the largest possible value that makes that statement true.

(c) Find the value of k.

[2 marks]